Introduction To Functional Analysis

Instructor: Professor Plamen Stefanov
Course Number: MA 54600
Credits: Three
Time: 3:00–4:15 PM TTh

Description

This course on Functional Analysis and will cover, as per the catalog description, the following topics: Fundamentals of functional analysis. Banach spaces, Hahn-Banach theorem. Principle of uniform boundedness. Closed graph and open mapping theorems. Applications. Hilbert spaces. Orthonormal sets. Spectral theorem for Hermitian operators and compact operators.

I will use the following book: M. Reed and B. Simon, Methods of Modern Mathematical Physics, vol.I: Functional Analysis. This is a beautiful and a highly regarded book aimed at mathematicians (and physicists) interested in applications of functional analysis to partial differential equations, and mathematical physics, just to name a few. Particular attention will be given to applications to differential and integral (linear) operators.

Introduction to Abstract Algebra

Instructor: Professor Freydoon Shahidi
Course Number: MA 55300
Credits: Three
Time: 10:30 – 11:20 AM MWF

Description

Pre–requisit: MA 503 or equivalent

Syllabus:

Group Theory: Review of basic definitions and facts including examples of groups: dihedral, symmetric, quaternions; isomorphism theorems, quotient groups, centralizers, normalizers, automorphisms.

Group actions on sets, orbits and stabilizers; representations of a group action as automorphisms of the set and its consequences: conjugation, class formula, p–groups, Sylow's theorem, composition series, solvable and nilpotent groups, simplicity of A_n for $n \geq 5$, direct and semi-direct products.

Ring Theory: Review of basic definitions and facts; isomorphism theorems, ideals, quotient rings, commutative rings: integral domains, maximal and prime ideals, PID, UFD, Euclidean domains, norms, some number theoretic applications: Fermat–Euler, Euler–Gauss theorems on sums of squares; Chinese remainder theorem with applications; ring of quotients for a domain; polynomial rings, reducibility criteria: Gauss's lemma, reduction criteria, Eisenstein polynomials and their irreducibility.

Field Theory: Field extensions, finitely generated and finite extensions, algebraic extensions, generating fields by roots of irreducible polynomials, separability, perfect fields, normal extensions, normal closures, splitting fields, finite fields, primitive elements and simple extensions, algebraically closed fields.

Galois Theory: Galois extensions, fundamental theorem of Galois theory, examples of Galois extensions, roots of unity, cyclotomic extensions, cyclic and abelian extensions, solvable extensions, cyclotomic polynomials, basic facts on Kummer extensions, extensions by radicals, non-solvability of polynomial equations of degree 5 and higher by radicals.

Book: D. Dummit and M. Foote, Abstract Algebra, John Wiley, 3rd Edition.

Abstract Algebra II
Instructor: Professor William Heinzer
Course Number: MA 55800
Credits: Three
Time: 3:30–4:20 PM MWF

Description

I plan to cover selected material developed in the book "Integral Domains Inside Noetherian Power Series Rings: Constructions and Examples" written by Christel Rotthaus, Sylvia Wiegand and me. The book is to be published this month (October 2021) in the AMS series Mathematical Surveys and Monographs . A preliminary version of the text is posted on my web page.

Algebraic K-theory
Instructor: Professor Deepam Patel
Course Number: MA 59800AK
Credits: Three
Time: 3:00–4:15 PM TTh

Description

This course will be an introduction to algebraic K-theory. The emphasis will be on explicitly computing algebraic K-theory, rather than building the abstract foundations. The course will likely be split into three parts. In the first part, we will discuss the Grothendieck group, Quillen's plus construction, Quillen's computation of the K-theory of finite fields, and Suslin's computation of K-theory of algebraically closed fields. In the second part, we will discuss the Quillen's Q-construction, and recall the foundational results on K-theory of exact categories due to Quillen. We will not prove these foundational results, but use them as a black box and apply them in order to perform various computations of K-theory of schemes. The goal of the second part will to give a proof of Gersten's conjecture in the geometric case (due to Quillen). In the last third of the of the course, I will discuss some more recent research oriented topics in K-theory. There are several directions we could pursue, and the topics covered will largely depend on the interests of the audience. Some potential topics include trace maps in K-theory, recent computations of p-adic K-theory via p-adic Hodge theory, descent theorems in algebraic K-theory, or construction of special elements in K-theory and their regulator computations (in particular, discussing Beilinson's conjectures on special values of L-functions). There are lots of open problems in the area (some of which are extremely difficult), and I hope to discuss at least some accessible problems which could serve as potential thesis problems.

References:: "Higher Algebraic K-theory: I" and "Higher Algebraic K- theory: II" by Quillen, "K-Book" by C. Weibel

Differential Topology
Instructor: Professor Ralph Kaufmann
Course Number: MA 59800DT
Credits: Three
Time: 12:00–1:15 PM TTh

Description

Differential topology is at the intersection of topology and analysis. One could say that the main aim is to derive topological data using differential calculus. This has the advantage that many notions become more intuitive. For instance, one can discuss cohomology using deRham forms and make Poincar duality explicit. One can also represent characteristic classes using forms. This can be of computational as well as of conceptual help and is essential in applications to differential geometry and physics, where topological invariants, such as charges, often come from integration.

Topics include: Differential forms on manifolds, Thom isomorphism, Poincare duality, Euler classes, coverings, Cech cohomology, spectral sequencs and the Cech-deRham isomorphism, bundles and characteristic classes.

We will use the classic text of Bott and Tu for the most part.

Representations of Lie Groups and Lie Algebras
Instructor: Professor Saugata Basu
Course Number: MA 59800RL
Credits: Three
Time: 3:30–4:20 PM MWF

Description

We will cover the basic theory of Lie groups and Lie Algebras and their representations (mostly over the complex numbers). Topics will include Cartan-Killing classification of semi-simple Lie algebras, root spaces and weight space theory, Lie algebra cohomology, Schur-Weyl duality, and connections with other areas such as Theoretical Computer Science (null cone membership problem, moment polytopes and connections with optimization). If time permits we will discuss recent work on $GL(\infty)$-invariant algebraic geometry and its connections with certain stability properties.

Wave Equations
Instructor: Professor Kiril Datchev
Course Number: MA 59800WE
Credits: Three
Time: 8:30–9:20 AM MWF

Description

In this course we will study wave equations, beginning with the simplest (D'Alembert's original) $u_{tt} = c^2 u_{xx}$ where the wavespeed c is a constant. We will then consider the Schrödinger equation $iu_t = -u_{xx}$ and the higher dimensional generalizations of both. We will derive solution formulas, travel speeds, the Huygens principle, propagation of singularities, and wave decay rates.

We will then use the methods of microlocal and semiclassical analysis, i.e. particle–wave duality, to examine analogs and generalizations of these results for other wave and Schrödinger equations, including allowing the wavespeed c to vary depending on the position and direction of the wave, and adding a potential energy term $V(x)u$ on the right hand side of the equation.

Spoiler alert: the general conclusion is the following. Behavior of solutions to the wave equation with wavespeed $c(x)$ is governed by the geometry of the trajectories of particles traveling with speed $c(x)$ according to Fermat's principle of least time. Behavior of solutions to the Schrödinger equation with potential energy $V(x)$ is governed by the geometry of the trajectories of particles traveling according to Newton's law $F = ma$ with force $F(x) = -V'(x)$. Both kinds of trajectories are described in a unified way by Hamilton's action principle, and the theories of microlocal and semiclassical analysis make the connection with wave evolution. More specifically, salient features of waves, especially singularities, follow these trajectories as they evolve.

As mentioned above, the simplest examples are waves which solve $u_{tt} = c^2 u_{xx}$. These waves take the form $u(x,t) = f(x+ct) + g(x-ct)$, where the f term corresponds to a particle traveling to the left at speed c, and the g term corresponds to a particle traveling to the right at speed c. In other examples we can usually describe neither the waves nor the particle trajectories so simply, but we instead relate more accessible major aspects of wave and particle behavior to one another. Especially important for waves are bound states, which live forever, and resonances, which can live for a long time. These correspond to particle trajectories which stay always in some bounded region.

For our work we will develop tools from distribution theory, Fourier analysis, Sobolev spaces, pseudodifferential operators, and scattering theory. These tools are also useful for the study of more general differential equations, and we will touch on such connections as we go. The course is intended to be accessible to students coming from a range of backgrounds and will assume knowledge only of real and complex analysis at the level of 440/504 and 425/525.

Sources for the material include Friedlander and Joshi's *Introduction to the Theory of Distributions*, Taylor's *Partial Differential Equations*, Zworski's *Semiclassical Analysis*, and Dyatlov and Zworski's *Mathematical Theory of Scattering Resonances*, but our treatment will be less advanced: notes will be available at the course website https://www.math.purdue.edu/ kdatchev/598/598.htm.

Numerical Methods For PDEs I

Instructor: Professor Xiangxiong Zhang
Course Number: MA 61500
Credits: Three
Time: 1:30–2:45 PM TTh

Description

This is an introductory course of numerical solutions to partial differential equations for any graduate students interested in computational mathematics, with emphasis on breadth rather than depth. The course will cover key concepts with a balance between analysis and implementation, including accuracy, stability and convergence of finite difference methods for time-dependent problems such as wave equatiodules of differentials encode derivations of commutative rings, and they describe singular loci and ramification loci.ns, parabolic equations and conservation laws. The finite element method for elliptic equations on structured meshes, which are equivalent to finite difference schemes, will also be introduced. Linear system solvers such as the conjugate gradient method and the multigrid method, and ODE solvers such as Runge-Kutta method will also be discussed, if time permits. Homework and the final exam will consist of both analysis and coding by Matlab. Sample Matlab codes will be provided thus prior knowledge of coding is not required. Recommended prerequisites include linear partial differential equations, linear algebra, and Fourier analysis, all of which will be reviewed during the lectures. Feel free to send an email to zhan1966@purdue.edu for questions. Last year's lecture notes are available at http://www.math.purdue.edu/∼zhan1966/teaching/615/MA615_notes.pdf However, this year's plan of lectures will be slightly different: nonlinear conservation laws will be discussed thoroughly.

Topics in Commutative Algebra: Duality Theory and Modules of Differentials

Instructor: Professor Bernd Ulrich
Course Number: MA 69000DM
Credits: Three
Time: 1:30–2:45 PM TTh

Description

The course will cover Gorenstein rings, canonical modules, local cohomology, local duality, modules of differentials and derivations. Gorenstein rings and canonical modules are the basic ingredients of a duality theory for modules over commutative rings, and local duality provides the connection with local cohomology, the algebraic version of sheaf cohomology. Modules of differentials encode derivations of commutative rings, and they describe singular loci and ramification loci.

The course serves in some sense as a continuation of MA 65000, but it is also independent from MA 65000 and should be accessible to somebody with basic knowledge of commutative algebra.

Prerequisites: Basic knowledge about commutative rings (such as the material of MA 55700).

Elliptic Curves

Instructor: Professor Daniel Le
Course Number: MA 69000EC
Credits: Three
Time: 10:30–11:45 AM TTh

Description

Elliptic curves are among the simplest "non-trivial" objects in subjects across a wide swath of mathematics including complex geometry, algebraic geometry, algebraic topology, and number theory. They correspondingly played a large role in the development of all of these. The course will begin with some analytic aspects of the theory before moving to arithmetic aspects. The focus of the course will be on the (co)homology of elliptic curves in its various guises (singular cohomology, de Rham cohomology, Tate modules, perhaps formal groups). To a degree, the topics will depend on the interests of the participants.