**Functions Of A Complex Variable I**
Instructor: Professor Kiril Datchev
Course Number: MA 53000
Credits: Three
Time: 8:30–9:20 AM MWF

**Description**

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics.

**Textbook:** "Introduction to Complex Analysis" by Michael E. Taylor. American Mathematical Society Graduate Studies in Mathematics #202, 2019.

**Elements Of Stochastic Processes**
Instructor: Professor Jing Wang
Course Number: MA 53200
Credits: Three
Time: 3:30–4:20 PM MWF

**Catalog Description**

A basic course in stochastic models, including discrete and continuous time Markov chains and Brownian motion, as well as an introduction to topics such as Gaussian processes, queues, epidemic models, branching processes, renewal processes, replacement, and reliability problems.

**Probability Theory I**
Instructor: Professor Rodrigo Banuelos
Course Number: MA 53800
Credits: Three
Time: 9:30–10:20 AM MWF

**Catalog Description**

Mathematically rigorous, measure-theoretic introduction to probability spaces, random variables, independence, weak and strong laws of large numbers, conditional expectations, and martingales.

**Ordinary Differential Equations And Dynamical Systems**
Instructor: Professor Aaron Yip
Course Number: MA 54300
Credits: Three
Time: 10:30–11:45 AM TTh

**PREREQUISITE:** One undergraduate course in each of the following topics:

- linear algebra (for example, MA 265, 351),
- differential equation (for example, MA 266, 366),
- analysis (for example, MA 341, 440, 504), or instructor's consent.

**Description**

This is a beginning graduate level course on ordinary differential equations. It covers basic results for linear systems, local theory for nonlinear systems (existence and uniqueness, dependence on parameters, flows and linearization, stable manifold theorem) and their global theory (global existence, limit sets and periodic orbits, Poincare maps). Some further topics include numerical methods, bifurcations, averaging techniques and applications to Hamiltonian mechanics and population dynamics.

Workload of this course consists of taking turns typing up lecture notes, a short paper (5 to 6 pages) and a final presentation of the paper.

**Textbook:** James D. Meiss: Differential Dynamical Systems (available online from Purdue)

**Real Analysis And Measure Theory**
Instructor: Professor Laszlo Lempert
Course Number: MA 54400
Credits: Three
Time: 4:30–5:20 PM MWF

**Catalog Description**

Metric space topology; continuity, convergence; equicontinuity; compactness; bounded variation, Helly selection theorem; Riemann-Stieltjes integral; Lebesgue measure; abstract measure spaces; $LP$-spaces; Holder and Minkowski inequalities; Riesz-Fischer theorem.

**Functions Of Several Variables And Related Topics**
Instructor: Professor Antonio Sa Barreto
Course Number: MA 54500
Credits: Three
Time: 4:30–5:45 PM TTh

**Description**

The Fourier (1768-1830) transform is a 19th century mathematical concept which is fundamental in 21st century mathematics and science in general. In this course we will study the Fourier transform in Euclidean space and its applications to partial differential equations. We plan to cover the following topics:

1. Review of measure theory, $L^p$ spaces and duality.
2. Rapidly decaying functions and their Fourier transform.
3. The inverse Fourier Tranform.
4. Plancherel Theorem and the Fourier transform as a linear operator acting on $L^2$.
5. Tempered distributions and their Fourier transform.
6. The Littlewood Payley decomposition.
7. Sobolev spaces and embedding theorems.
8. Algebras of functions.
9. Introduction to Pseudodifferential Operators in $\mathbb{R}^n$.

10. Elliptic partial differential equations.
11. Almost orthogonality and the Cotlar-Stein Lemma.
12. The Calderón-Vaillencourt Theorem.
13. Introduction to the Paradifferential Calculus and applications to nonlinear elliptic equations after De Giorgi, J. Nash, J. Moser, etc.
14. The Thomas-Stein Restriction Theorem and applications to partial differential equations.

**Textbook:** No textbook required. I will write my own lecture notes and post them in Brightspace. I will also post my handwritten class notes in Brightspace.

**Suggested books and articles:** See references [1, 2, 3].

**Grade:** Homework will be assigned.

**REFERENCES**


[1] Alinhac, Serge; Gérard, Patrick. *Pseudo-differential operators and the Nash-Moser theorem.* Graduate Studies in Mathematics, 82. American Mathematical Society, Providence, RI, 2007. viii+168 pp. ISBN: 978-0-8218-3454-1

[2] Meyer, Yves. *Remarques sur un théorème de J.-M. Bony.* Proceedings of the Seminar on Harmonic Analysis (Pisa, 1980). Rend. Circ. Mat. Palermo (2) 1981, no. suppl, suppl. 1, 1-20.

[3] Stein, Elias M. *Singular integrals and differentiability properties of functions.* Princeton Mathematical Series, No. 30 Princeton University Press, Princeton, N.J. 1970 xiv+290 pp.


## Introduction To Functional Analysis

Instructor: Professor Andrew Toms
Course Number: MA 54600
Credits: Three
Time: 9:30–10:20 AM MWF

### Description

Nets and convergence, Zorn's Lemma, Banach spaces, dual spaces, the Hahn-Banach Theorem(s), bounded, compact, and Fredholm operators on Hilbert space, culminating in various versions of the spectral theorem. Text will be G. K. Pedersen's Analysis Now, a delightfully action-packed and inexpensive tome.

## Introduction To Abstract Algebra

Instructor: Professor Bernd Ulrich
Course Number: MA 55300
Credits: Three
Time: 3:30–4:20 PM MWF

### Catalog Description

Group theory: Sylow theorems, Jordan-Holder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

**Linear Algebra**
Instructor: Professor Saugata Basu
Course Number: MA 55400
Credits: Three
Time: 1:30–2:20 PM MWF

**Catalog Description**

Review of basics: vector spaces, dimension, linear maps, matrices determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

**Abstract Algebra II**
Instructor: Professor Daniel Le
Course Number: MA 55800
Credits: Three
Time: 12:30–1:20 PM MWF

**Description**

This course is an introduction to representation theory following Representation Theory: A First Course by Fulton and Harris. Representation theory is an indispensable tool in many different areas. We will focus on examples from finite groups and Lie algebras. The prerequisites are group theory and linear algebra (including multilinear algebra and tensor products).

**Introduction In Algebraic Topology**
Instructor: Professor Jeremy Miller
Course Number: MA 57200
Credits: Three
Time: 12:00–1:15 PM TTh

**Description**

Math 572 covers homology (and its variant cohomology) which gives an algebraic measurement of the number and kind of "holes" in a topological space. Homology is a more computable version of the fundamental group (covered in 571) and its higher generalizations. It is a ubiquitous notion that appears in far ranging subjects such as complex analysis and arithmetic geometry. We will discuss applications of homology such as to fixed-point problems. We will review some basic homological algebra such as Tor groups. Time permitting, we will cover most of chapters 2 and 3 of Hatcher's *Algebraic Topology*.

**Graph Theory**
Instructor: Professor Giulio Caviglia
Course Number: MA 57500
Credits: Three
Time: 12:00–1:15 PM TTh

**Catalog Description**

Introduction to graph theory with applications.

**Algebraic Geometry II**
Instructor: Professor Deepam Patel
Course Number: MA 59800AG
Credits: Three
Time: 12:00–1:15 PM TTh

**PREREQUISITE:** First semester course in AG (roughly Chapter 1 of Hartshorne, and some of Chapter 2 including basics of schemes and quasi-coherent sheaves).

**Description**

This course will be a continuation of the first semester AG course. In particular, we will begin with the study of cohomology of (quai-coherent) sheaves, leading up to Serre duality and Riemann-Roch for curves. I anticipate this will take roughly 6-8 weeks (though perhaps longer or shorter depending on how much is covered during the Fall course). For the remainder of the course, we will cover one or more (depending on time) of the following topics based on audience interests:This is to let you know that I have approved this case.

(1) Grothendieck-Riemann Roch
(2) Classification of Algebraic Surfaces
(3) Grothendeick Duality
(4) An introduction to intersection theory and enumerative geometry

**Introduction to the circle method and its application**
Instructor: Professor Trevor Wooley
Course Number: MA 59800ANT
Credits: Three
Time: 12:00–1:15 PM TTh

**PREREQUISITE:** Elementary number theory and basic analysis

**Description**

This course serves as an introduction to analytic number theory via the (Hardy-Littlewood) circle method. Background results from number theory and harmonic analysis will be reviewed as needed. Students already familiar with the basic elements of the circle method will acquire knowledge of more advanced topics, such as the use of smooth numbers, and the delta-function formulation of the method.

The (Hardy-Littlewood) circle method applies Fourier analysis to count rational or integral solutions of an equation or inequality in a manner respecting the inherent arithmetic. Developments in recent years have broadened its impact into additive combinatorics and discrete harmonic analysis beyond its more traditional role in quantitative arithmetic geometry.

We shall take as our central example Waring's problem – the problem of understanding the number of representations of an integer as the sum of a fixed number of $k$-th powers of positive integers. Our aims are twofold: (i) to understand the scope and limitations of the circle method, and (ii) to gain some facility to apply the method, so from time to time there will be technical material that we'll just cite rather than prove in any detail. This course is intended to be accessible to those without any background in analytic number theory, and to provide an introduction to some basic ideas in analytic number theory.

**Assessment:** Six problem sets will be offered through the semester, and class participants can demonstrate engagement with the course by any written and/or in-class presentations featuring a reasonable subset of these problems – three levels of difficulty: short problems testing basic skill-sets, extended problems integrating the essential methods of the course, and more challenging problems for enthusiasts with detailed hints available on request.

**Contents:**

(i) Discussion of Weyl's inequality, Hua's Lemma, and the simplest treatment of Waring's problem. This provides an opportunity to discuss the key elements of the major arc analysis, that is, the singular integral and singular series, that together constitute the product of local densities.

(ii) Smooth numbers – integers all of whose prime divisors are small – and their application in the circle method. Basic properties of the smooth numbers, including their distribution in arithmetic progression. Mean value estimates via efficient differencing, and new estimates of Weyl type.

(iii) Application of smooth Weyl sums to equidistribution modulo 1, including fractional parts problems, and to upper bounds for the quantity $G(k)$ in Waring's problem.

(iv) The delta function variant of the circle method and its applications.

The course will be based on the instructor's lecture notes. Good texts for background reading and support are: This is to let you know that I have approved this case.

- R. C. Vaughan, The Hardy-Littlewood method, 2nd edn., Cambridge Tract No. 125, Cambridge University Press, 1997 [Condensed, but the best source in print; updated from the 1981 first edition.]

- H. Davenport, Analytic methods for Diophantine equations and Diophantine inequalities, Ann Arbor Publishers, Ann Arbor, 1962 or the LaTeXed version published by Cambridge University Press in 2005 [Friendlier for the basics, with material on general homogeneous cubics, but misses modern developments.]

- M. Nathanson, Additive number theory. The classical bases, GTM 164, Springer-Verlag, New York, 1996 [Pedestrian approach to the basics in which no corner is cut – good for getting started!]

**Introduction To Mathematical Biology**
Instructor: Professor Alexandria Volkening
Course Number: MA 59800BM
Credits: Three
Time: 10:30–11:45 AM TTh

## Description

This course will introduce participants to mathematical biology with a mathematical modeling-centric perspective. We will discuss several research vignettes, including examples from epidemiology and developmental biology, and use biological questions to illustrate both classic approaches and emerging techniques. For example, we will discuss compartmental modeling, dynamics on and of networks, parameter estimation, reaction-diffusion equations, cellular automaton modeling, and agent-based modeling. We will also highlight how data-driven methods for equation learning and topological data analysis (especially persistent homology) are being used in new ways to address challenges in mathematical biology now.

Complementing this, we will talk about methods for reading biological papers, working with quantitative or qualitative data, and effectively communicating mathematics in written and oral form across disciplinary boundaries. We will use computation, analysis, and modeling. Throughout the course, we will point out biology–math feedback loops, looking for how math can suggest experiments and how taking a biological perspective can drive new math. The latter portion of this course will include student presentations and discussions of research papers, and students will each complete a mini research project.

**Other notes:** There will be no exams, and grades will be based on participation, presentations, a few homework assignments, and the mini research project. In terms of background, experience with linear algebra and differential equations at the undergraduate level will be assumed; I anticipate that most projects will involve some simulations.

**Topics on optimization algorithms**
Instructor: Professor Xiangxiong Zhang
Course Number: MA 59800CO
Credits: Three
Time: 9:30–10:20 AM MWF

## Description

This is a topic course on optimization algorithms with emphasis on the analysis. Prior knowledge and experience of numerical optimization algorithms is preferred but not required. The first two weeks will be an introduction of basic knowledge in numerical optimization such as line search methods and the convergence. Then I will spend at least 2-3 weeks on each of the following topics of popular large scale optimization algorithms:

1. Superlinear methods for smooth functions such as the conjugate gradient method and L-BFGS method.
2. The convex optimization algorithms for nonsmooth functions such as TV minimization and ell-1 minimization, including the ADMM, primal-dual method, and Douglas-Rachford splitting.
3. Nesterov's convergence analysis and Nesterov's acceleration method.
4. Stochastic gradient descent method.

If time permits, Riemannian optimization will also be briefly introduced. No homework will be assigned, but class attendance will be enforced. A final report and presentation of reading one or more papers on one particular topic will be required.

**Category theory and simplicial methods**
Instructor: Professor Manuel Rivera
Course Number: MA 59800CT
Credits: Three
Time: 9:00–10:15 AM TTh

## Description

This course will be an introduction to category theory with emphasis on simplicial methods and their use in topology, homological algebra, and algebraic geometry. In the first part, we begin by carefully discussing the basic concepts of category theory together with many examples from different fields of mathematics. These include categories, functors, natural transformations, universal properties, limits and colimits, adjunctions, monads, and Kan extensions. In the second part, we focus on developing the basics of simplicial homotopy theory and discuss "derived" versions of some of the constructions introduced in the first part of the course. We will end with a few glimpses of higher category theory. This course should be accessible to a relatively wide range of graduate students with different background and interests. Some familiarity with basic algebraic topology will be helpful. In the first part of the course we will follow the textbooks "Category theory in context" by Emily Riehl and "Categories for the working mathematician" by Saunders MacLane supplemented by more examples and applications. In the second part, we follow some of "Categorical homotopy theory" by Riehl, parts of "Methods of homological algebra" by Gelfand and Manin, and parts of "Simplicial homotopy theory" by Goerss and Jardine.

**Radon Transforms**
Instructor: Professor Plamen Stefanov
Course Number: MA 59800RT
Credits: Three
Time: 4:30–5:45 PM TTh

## Description

The Radon transform $Rf$ maps a function $f$ to its integrals along all (hyper-)planes. The associated X-ray transform $Xf$ integrates $f$ along all lines. A fundamental problem is the inversion of those transforms in various situations: with full data (there are explicit formulas then), with incomplete, respectively discrete data, in presence of noise, etc. Studied first by Radon, and rediscovered by A. Cormack and G. Hounsfield (the 1979 Nobel prize in Physiology and Medicine), the inversion of the X-ray transform is the mathematical model of CT (Computed Tomography) scan, also known as CAT scan. More general transforms, like the X-ray transform over geodesics of a certain metric appear in various applications, for example in seismology, and is of its own interest in geometry.

We will start and stay mostly with the Euclidean case. The first part of the course will study the mapping properties of $R$ and $X$, extension to distributions (which I will briefly introduce for those not familiar with them), inversion formulas, stability estimates, range conditions, support theorems, recovery in a region of interest with incomplete data. We will study the X-ray transforms of tensor fields, as well, and explain the motivation. If time permits, I will introduce the light-ray transform: integrals of functions $f(t, x)$ over light-rays in the Minkowski metric, and discuss its invertibility.

The second part of the course will concentrate on the weighted X-ray transform and microlocal considerations. I will introduce some microlocal concepts briefly and explain what they predict about recovery of singularities (e.g., edges) with incomplete data, in particular. Numerical examples will be presented.

The course should be accessible to students having good analysis background, including some familiarity with functional analysis (Hilbert spaces, linear operators but no deep knowledge is required), and the Fourier transform. I will follow a book by me and G. Uhlmann which I will make available online (an older version is on my website even now). This book is still not finished but the part needed for this course is complete. Relevant books for the first part of the course are also the classical book by Helgason "Radon Transform", available for free on his webpage, and Natterer's book "The Mathematics of Computerized Tomography".

## Finite Element Methods for Partial Differential Equations

Instructor: Professor Zhiqiang Cai
Course Number: MA 61500
Credits: Three
Time: 1:30–2:45 PM TTh

**PREREQUISITE:** MA/CS 514 or equivalent or consent of instructor

### Description

The finite element method is the most widely used numerical technique in computational science and engineering. This course covers the basic mathematical theory of the finite element method for partial differential equations (PDEs) including variational formulations of PDEs and construction of continuous finite element spaces. Adaptive finite element method as well as fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented. When time permits, neural network as a new class of approximating functions will also be covered.

### References


[1] S. Brenner and R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, New York, 2002.

[2] D. Braess, Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics, Cambridge University Press, New York, 1997.

[3] C. Johnson, Numerical Solution of Partial differential Equations by the Finite Element Method, Cambridge University Press, Cambridge, 1987.


## Methods Of Linear And Nonlinear Partial Differential Equations II

Instructor: Professor Isaac Harris
Course Number: MA 64300
Credits: Three
Time: 12:00–1:15 PM TTh

### Description

Continuation of MA 642. Topics to be covered are Lp theory for solutions of elliptic equations.

Introduction to evolution problems for parabolic and hyperbolic equations including Galerkin approximation. Applications to nonlinear problems. Here are some topics:

-Linear and Quasi-Linear BVPs

-Boundary Integral Equations

-Basics of Galerkin Methods

-Well-posedness for Evolution Equations

-Applications to Inverse Problems.

**Evaluation:** The grade will be the average homework assignments given periodically throughout the semester via Gradescope. All questions will come from topics covered in the lecture.

**Reference Texts(optional):**

-Partial Differential Equations in Action: From Modeling to Theory by S. Salsa

-Variational Techniques for Elliptic Partial Differential Equations by F. Sayas, T. Brown and M. Hassell

-Strongly Elliptic Systems and Boundary Integral Equations by W. McLean

-Linear Integral Equations by R. Kress

## Class Field Theory

Instructor: Professor Freydoon Shahidi
Course Number: MA 68400
Credits: Three
Time: 10:30–11:20 PM MWF

### Description

Class field theory is the study of abelian extensions (finite or infinite) of local and global fields by means of closed subgroups of idele class group through the Artin reciprocity map. It is a crowning achievement of number theory in the 20th centry which is still quite influencial in many related fields including arithmetic geometry, analytic number theory and automorphic forms. In fact, Langlands program has been an effort to extend (abelian) class field theory to the mysterious non-abelian setting, including suitable generalizations of Artin reciprocity law.

**Syllabus:** Ideles, adeles, Hecke $L$-functions and their continuation, first and second inequalities of class field theory with related cohomology theory, Artin symbol and reciprocity law, local and global class fields, Kronecker–Weber theorem.

I will generally follow my notes which are posted on my bio page and available on internet.

Other useful sources are:

1. S. Lang, “Algebraic Number Theory”
2. J. Neukirch, “Algebraic Number Theory”
3. J.W. Cassels and A. Frolich, “Algebraic Number Theory”

The course has no exams. There will be homework assignments which will be graded and will provide your course grade.