Quiz 6

MA 262
Artur's Class

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Problem 1

Let S be the subspace of $\mathbb{R}^3$ consisting of all solutions to the equation

$$2x - y - 3z = 0.$$

Give a set of vectors which span S. (How many vectors do you need?)

Solution

The geometry: this is a 2 dimensional plane sitting in 3-space. We need only 2 vectors to span it. (Also we need two non-colinear vectors, i.e., vectors which are not multiples of eachother.) So we'll try in any way possible to get them. The easiest way is as follows.

Solve for one variable. In this case, I'll choose y, because it has no coefficient, allowing us to avoid any inessential arithmetic. We get $y = 2x - 3z$. Now try plugging in 'easy' numbers into the right hand side. For example, $x = 0, z = 1$, and $x = 1, z = 0$. This gives us two vectors:

$$(1, 2, 0),\ (0, -3, 1).$$

Problem 2

Consider the vector space, $M_2(\mathbb{R})$ of 2×2 matrices of real numbers. Do the following two matrices span $M_2(\mathbb{R})$?

$$\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix},\ \ \mathbf{N} = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix},$$

Show work and/or say something interesting about why this is obviously true or obviously false. Convince me, but keep it short and precise!

Solution

This problem is easy! The quick observation is that the space is in all respects the same as $\mathbb{R}^4$, i.e., it is just 4-tuples of numbers (vectors) but written in a square. If someone asked you if 2 vectors span all of $\mathbb{R}^4$, you should look at them like they are crazy.

Thought about it differently: For any (x, y, z, w) one must solve the linear equation

$$a\mathbf{u} + b\mathbf{v} = (x, y, z, w),$$

where $\mathbf{u}$ and $\mathbf{v}$ are 4-vectors formed from $\mathbf{M}$ and $\mathbf{N}$ above. This is of course impossible.

Note: If I gave you 0, 1, 2, or 3 matrices, this question is trivial. It only requires any work if I give you 4 matrices. (This is all because the space $M_2(\mathbb{R})$ is 4-dimensional.