Quiz 2 — Solutions

MA 262
Artur's Class

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Problem 1

Find a nonconstant solution $y = y(x)$ of

$$\frac{dy}{dx} = \frac{2}{3}(y-1)^{1/2}$$

satisfying $y(1) = 1$. This solution is defined over which domain?

Solution

Rearrange and apply integrals to get

$$\frac{1}{2}\int \frac{dy}{(y-1)^{1/2}} = \frac{1}{3}\int dx$$

Performing the integration gives

$$\sqrt{y-1} = \frac{1}{3}x^2 + c,$$

where c is some constant of integration. At this point substituting the initial data, gives $0 = 1/3 + c$, i.e., $c = -1/3$. Squaring and rearranging gives

$$y(x) = \frac{1}{9}(x^2-1)^2 + 1.$$

Thinking of the solution $y = y(x)$ as a function $y : \mathbb{R} \to \mathbb{R}$, we can see that this solution is defined over the entire real line. (Notice also that this solutions satisfies $y(x) \geq 1$ for all real x. So this causes no problem with the radical in the differential equations.)

Problem 2

Use an integrating factor to find a solution $y = y(x)$ of

$$(y - e^x)\ dx + dy = 0$$

satisfying $y(1) = 1$. Denote clearly which integrating factor you used.

Solution

Put the equation in in standard form:

$$\frac{dy}{dx} + y = e^x.$$

The coefficient of y is the constant function $p(x) \equiv 1$. The integrating factor $\mu(x) = \exp \int p(x)\ dx = e^x$. Multiplying by $\mu = e^x$ gives

$$\frac{d}{dx}(e^x y) = e^x \frac{dy}{dx} + e^x y = e^{2x}$$

Integrating gives

$$e^x y = \int e^{2x}\ dx = \frac{1}{2}e^{2x} + c,$$

for some constant c. Substituting in the intial data gives $e = \frac{1}{2}e^2$, i.e., $c = e - \frac{1}{2}e^2$. Multiplying both sides by e^{-x} gives

$$y(x) = \frac{1}{2}e^x + ce^{-x} = \frac{1}{2}e^{2x} + (e - \frac{1}{2}e^2)e^{-x}.$$

NB: Notice that multiplying a constant c by e^{-x} does not result in a constant. In other words we cannot write $ce^{-x} = c'$ for some new constant c'.

So don't do that :)