Quiz 4

MA 262
Artur's Class

February 14, 2012

Problem 1

Put

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Compute the commutator $[B, C] := BC - CB$.

Solution

First compute both products

$$BC = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad CB = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

So their difference is

$$BC - CB = \begin{pmatrix} 0 & +2 \\ -2 & 0 \end{pmatrix}.$$

Problem 2

Put

$$A = \begin{pmatrix} 3 & -1 \\ -5 & -1 \end{pmatrix}.$$

Compute A^2.

Solution

$$A^2 = A \cdot A = \begin{pmatrix} 14 & -2 \\ -10 & 6 \end{pmatrix}.$$

Problem 3

With A as above, what is $A^2 \cdot A - A \cdot A^2$?

Solution

Stop and think about this for a second. For any square matrix A we have $A^2 \cdot A = A^3 = A \cdot A^2$ and hence $A^2 \cdot A - A \cdot A^2 = A^3 - A^3 = \mathbf{0}$. So that means the above expression is the 2×2 matrix of zeros. This doesn't require any computation. :)

Problem 4

Compute the reduced row echelon from (RREF) of the following matrix.

$$M = \begin{pmatrix} 2 & -1 \\ 3 & 2 \\ 2 & 5 \end{pmatrix}.$$

What is its rank?

Solution

This was a homework problem from the book. So I won't write out the lengthy process of row reduction. The book has a good section on it. I will however show you a dirty trick:

Recall that the rank of a matrix is the dimension of the image of the associated operator. Notice this matrix can be thought of as a linear operator $\mathbb{R}^2 \to \mathbb{R}^3$. This can have either rank 1 or 2. Just look at the images of the canonical basis vectors

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The images of these under the map, are precisely the columns of M. Notice that the columns are linearly independent. (One is not the scalar multiple of the other.) This means that the map has a 2 dimensional image, i.e., it has full rank. Thus a row echelon form of the matrix would be of the form

$$\begin{pmatrix} 1 & x \\ 0 & 1 \\ 0 & 0 \end{pmatrix},$$

for some $x \in \mathbb{R}$. (That's how your book defined the rank of a matrix on page 146.) And you could do an elementary row operation on this matrix to get the RREF

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

If this doesn't make much sense now, then no worries. You will understand every bit of it by the time we get into some deeper linear algebra. And just to be sure, none of this knowledge was required to compute the RREF form of that matrix. (This was just a sneaky way for me to compute so I didn't have to go through all the error prone row reduction steps.)