Quiz 5

MA 262
Artur's Class

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Problem 1

Put

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Compute $\det(A)$

Solution

This matrix is upper triangular (UT). So we can compute along the diagonal:

$$\det(A) = 1 \cdot 5 \cdot 8 \cdot 1 = 40.$$

(This is an immediate consequence of the cofactor expansion method.)

Problem 2

Recall $C(\mathbb{R})$ is the real vector space of continuous functions on $\mathbb{R}$. The polynomials of degree ≤ 2 form a subset. Show that this is also a subspace.

Solution

We can use the subspace criteria: (i) nonemptyness, (ii) closed under addition, and (iii) closed under scalar multiplication.

- *Nonempty.* The zero polynomial $0 = 0 + 0x + 0x^2$ is of degree two or less. So at least the zero polynomial is in our set.
- *Closed under* $+$. Let $f(x) = a_0 + a_1x + a_2x^2$ and $g(x) = b_0 + b_1x + b_2x^2$ by polynomials of degree 2 or less. Then their sum

  $$(f + g)(x) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2$$

  is clearly a polynomial of degree 2 or less.
- *Closed under* $\cdot$. Let $f(x) = a_0 + a_1x + a_2x^2$ and k be a scalar (i.e., $k \in \mathbb{R}$). Again, it is obvious that

  $$(k \cdot f)(x) = ka_0 + ka_1x + ka_2x^2,$$

  is a polynomial of degree 2 or less.

Remark. In all of the above, even though the polynomials "look" degree 2, it is possible that the coefficient of x^2 is zero. In this case the degree is strictly less than 2.

Problem 3

What about for polynomials of degree = 2? Explain.

Solution

There are many ways to see that the set of degree 2 polynomials is **not** a subspace. Here are a few methods:

- This space has no zero! (The zero polynomial is not a degree two polynomial.)
- It is not closed under scalar multiplication. $0 \cdot p(x) = 0$. But again the zero polynomial is not of degree 2.
- It is not closed under addition. Consider a degree two polynomial $p(x)$ and its negative $-p(x)$. Their sum is zero and hence not of degree 2.