Quiz 6 — Solutions

MA 262
Artur's Class

February 29, 2012

Problem 1

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

Compute nullspace(A).

Solution

Recall that the nullspace of A is the set of vectors $v \in \mathbb{R}^2$ that are mapped to 0 under A, i.e., $Av = 0$. Suppose $v = (x_1, x_2)$ is in the nullspace of A. Then $(0, 0) = Av = (2x_1, 0)$. Notice only x_1 is constrained. The nullspace is then

$$\text{nullspace}(A) = \{(0, r) : r \in \mathbb{R}\}.$$

Problem 2

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Compute nullspace(A).

Solution

This problem is easy since every vector in $\mathbb{R}^2$ is "killed by A," i.e., $Av = 0$ for all $v \in \mathbb{R}^2$. Thus

$$\text{nullspace}(A) = \mathbb{R}^2.$$

Problem 3

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

Compute nullspace(A).

Solution

This problem is also easy because it is clear that $\text{rank}(A) = 2$; so A has full rank and is thus nonsingular. That means that the equation $Av = 0$ has no nontrivial solutions. Thus the nullspace is the trivial (zero) subspace:

$$\text{nullspace}(A) = \{(0,0)\}.$$

Problem 4

Consider the differential equation

$$y'' + 2y' - y = 1.$$

(a) Write down the solution space in set notation. (Do not solve the equation.)

(b) Is this solution space a subspace of $C(\mathbb{R})$.

Solution

(a) The solution space is

$$S = \{y \in C(\mathbb{R}) : y'' + 2y' - 1 = 0\}.$$

(b) To see that $S \subset C(\mathbb{R})$ is not a subspace of $C(\mathbb{R})$ we can quickly observe that this subset doesn't contain the zero (the zero function $O(x) \equiv 0$) of $C(\mathbb{R})$.