SOCIETY OF ACTUARIES

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE QUESTIONS

Interest Theory

This page indicates changes made to Study Note FM-09-05.

January 14, 2014:

Questions and solutions 58–60 were added.

June, 2014

Question 58 was moved to the Derivatives Markets set of sample questions.

Questions 61-73 were added.

December, 2014: Questions 74-76 were added.

Many of the questions were re-worded to conform to the current style of question writing. The substance was not changed.

Some of the questions in this study note are taken from past SOA/CAS examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

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1.

Bruce deposits 100 into a bank account. His account is credited interest at an annual nominal rate of interest of 4% convertible semiannually.

At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at an annual force of interest of δ .

After 7.25 years, the value of each account is the same.

Calculate δ .

(A) 0.0388

(B) 0.0392

(C) 0.0396

(D) 0.0404

(E) 0.0414

2.

Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i.

The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years.

Calculate X.

(A) 4695

(B) 5070

(C) 5445

(D) 5820

(E) 6195

3.

Eric deposits 100 into a savings account at time 0, which pays interest at an annual nominal rate of *i*, compounded semiannually.

Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of *i*.

Eric and Mike earn the same amount of interest during the last 6 months of the 8^{th} year.

Calculate *i*.

(A) 9.06%

(B) 9.26%

(C) 9.46%

(D) 9.66%

(E) 9.86%

4.

John borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1,627.45 at the end of each year. Instead, John repays the 10,000 using a sinking fund that pays an annual effective interest rate of 14%. The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan and are made at the end of each year for 10 years.

Calculate the balance in the sinking fund immediately after repayment of the loan.

(A) 2,130

(B) 2,180

(C) 2,230

(D) 2,300

(E) 2,370

5.

An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31.

Calculate the dollar-weighted (money-weighted) rate of return for the year.

(A) 9.0%

(B) 9.5%

(C) 10.0%

(D) 10.5%

(E) 11.0%

6.

A perpetuity costs 77.1 and makes end-of-year payments. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, …., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%.

Calculate n.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

7.

1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted.

The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%.

Calculate the accumulated value of Fund Y at the end of year 10.

(A) 1519

(B) 1819

(C) 2085

(D) 2273

(E) 2431

8. Deleted

9.

A 20-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is X.

The lender charges interest at an annual effective rate of 10%.

Calculate X.

(A) 32

(B) 57

(C) 70

(D) 97

(E) 117

10.

A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%.

Calculate the interest portion of the 7th coupon.

(A) 632

(B) 642

(C) 651

(D) 660

(E) 667

11.

A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment.

The annual effective rate of interest is 8%.

Calculate X.

(A) 54

(B) 64

(C) 74

(D) 84

(E) 94

12.

Jeff deposits 10 into a fund today and 20 fifteen years later. Interest for the first 10 years is credited at a nominal discount rate of d compounded quarterly, and thereafter at a nominal interest rate of 6% compounded semiannually. The accumulated balance in the fund at the end of 30 years is 100.

Calculate d.

(A) 4.33%

(B) 4.43%

(C) 4.53%

(D) 4.63%

(E) 4.73%

13.

Ernie makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest

$\delta_t = \frac{t^2}{100}$, $t > 0$.

The amount of interest earned from time 3 to time 6 is also X.

Calculate X.

(A) 385

(B) 485

(C) 585

(D) 685

(E) 785

14.

Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the payment in that year is K% larger than the payment in the year immediately preceding that year, where $K < 9.2$.

At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50.

Calculate K.

(A) 4.0

(B) 4.2

(C) 4.4

(D) 4.6

(E) 4.8

15.

A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective interest rate of 8.07%.

(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Calculate i.

(A) 8.75%

(B) 9.00%

(C) 9.25%

(D) 9.50%

(E) 9.75%

16.

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40th payment is made.

(A) 6750

(B) 6890

(C) 6940

(D) 7030

(E) 7340

17.

To accumulate 8000 at the end of 3*n* years, deposits of 98 are made at the end of each of the first *n* years and 196 at the end of each of the next 2*n* years.

The annual effective rate of interest is *i*. You are given $(1+i)^n = 2.0$.

Calculate *i*.

(A) 11.25%

(B) 11.75%

(C) 12.25%

(D) 12.75%

(E) 13.25%

18.

Olga buys a 5-year increasing annuity for X.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.

The annual nominal interest rate is 9% convertible quarterly.

Calculate X.

(A) 2680

(B) 2730

(C) 2780

(D) 2830

(E) 2880

19.

You are given the following information about the activity in two different investment accounts:

Account K			
	Fund value	Activity	
Date	before activity	Deposit	Withdrawal
January 1, 2014	100.0		
July 1, 2014	125.0		X
October 1, 2014	110.0	$2X$	
December 31, 2014	125.0		

Account L			
	Fund value	Activity	
Date	before activity	Deposit	Withdrawal
January 1, 2014	100.0		
July 1, 2014	125.0		X
December 31, 2014	105.8		

During 2014, the dollar-weighted (money-weighted) return for investment account K equals the time-weighted return for investment account L, which equals i.

Calculate i.

(A) 10%

(B) 12%

(C) 15%

(D) 18%

(E) 20%

20.

David can receive one of the following two payment streams:

(i) 100 at time 0, 200 at time n years, and 300 at time $2n$ years

(ii) 600 at time 10 years

At an annual effective interest rate of i, the present values of the two streams are equal.

Given $v^n = 0.76$, calculate i.

(A) 3.5%

(B) 4.0%

(C) 4.5%

(D) 5.0%

(E) 5.5%

21.

Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$.

Interest is credited at a force of interest $\delta_t = \dfrac{1}{8+t}$.

After time 10, the account is worth 20,000.

Calculate k.

(A) 111

(B) 116

(C) 121

(D) 126

(E) 131

22.

You have decided to invest in Bond X, an *n*-year bond with semi-annual coupons and the following characteristics:

(i) Par value is 1000.

(ii) The ratio of the semi-annual coupon rate, r, to the desired semi-annual yield rate, i, is 1.03125.

(iii) The present value of the redemption value is 381.50.

Given $(1+i)^{-n} = 0.5889$, calculate the price of bond X.

(A) 1019

(B) 1029

(C) 1050

(D) 1055

(E) 1072

23.

Project P requires an investment of 4000 today. The investment pays 2000 one year from today and 4000 two years from today.

Project Q requires an investment of X two years from today. The investment pays 2000 today and 4000 one year from today.

The net present values of the two projects are equal at an annual effective interest rate of 10%.

Calculate X.

(A) 5400

(B) 5420

(C) 5440

(D) 5460

(E) 5480

24.

A 20-year loan of 20,000 may be repaid under the following two methods:

(i) amortization method with equal annual payments at an annual effective interest rate of 6.5%

(ii) sinking fund method in which the lender receives an annual effective interest rate of 8% and the sinking fund earns an annual effective interest rate of j

Both methods require a payment of X to be made at the end of each year for 20 years.

Calculate j.

(A) 6.4%

(B) 7.6%

(C) 8.8%

(D) 11.2%

(E) 14.2%

25.

A perpetuity-immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and a charity receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and the charity's share is K.

Calculate K.

(A) 24%

(B) 28%

(C) 32%

(D) 36%

(E) 40%

26.

Seth, Janice, and Lori each borrow 5000 for five years at an annual nominal interest rate of 12%, compounded semi-annually.

Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.

Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

(A) 8718

(B) 8728

(C) 8738

(D) 8748

(E) 8758

27.

Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X.

Calculate X.

(A) 28.00

(B) 31.30

(C) 34.60

(D) 36.70

(E) 38.90

28.

Ron is repaying a loan with payments of 1 at the end of each year for n years. The annual effective interest rate on the loan is i. The amount of interest paid in year t plus the amount of principal repaid in year $t + 1$ equals X.

Determine which of the following is equal to X.

(A) $1+\dfrac{v^{n-t}}{i}$

(B) $1+\dfrac{v^{n-t}}{d}$

(C) $1+v^{n-t}i$

(D) $1+v^{n-t}d$

(E) $1+v^{n-t}$

29.

At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 3, is 32.

At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months, is X.

Calculate X.

(A) 31.6

(B) 32.6

(C) 33.6

(D) 34.6

(E) 35.6

30.

As of 12/31/2013, an insurance company has a known obligation to pay 1,000,000 on 12/31/2017. To fund this liability, the company immediately purchases 4-year 5% annual coupon bonds totaling 822,703 of par value. The company anticipates reinvestment interest rates to remain constant at 5% through 12/31/2017. The maturity value of the bond equals the par value.

Consider two reinvestment interest rate movement scenarios effective 1/1/2014. Scenario A has interest rates drop by 0.5%. Scenario B has interest rates increase by 0.5%.

Determine which of the following best describes the insurance company's profit or (loss) as of 12/31/2017 after the liability is paid.

(A) Scenario A – 6,610, Scenario B – 11,150

(B) Scenario A – (14,760), Scenario B – 14,420

(C) Scenario A – (18,910), Scenario B – 19,190

(D) Scenario A – (1,310), Scenario B – 1,320

(E) Scenario A – 0, Scenario B – 0

31.

An insurance company has an obligation to pay the medical costs for a claimant. Average annual claims costs today are 5000, and medical inflation is expected to be 7% per year. The claimant is expected to live an additional 20 years.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.

Calculate the present value of the obligation using an annual effective interest rate of 5%.

(A) 87,900

(B) 102,500

(C) 114,600

(D) 122,600

(E) Cannot be determined

32.

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

(A) -1398

(B) -699

(C) 699

(D) 1398

(E) 2,629

33.

You are given the following information with respect to a bond:

(i) par value: 1000

(ii) term to maturity: 3 years

(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

Calculate the value of the bond.

(A) 906

(B) 926

(C) 930

(D) 950

(E) 1000

34.

You are given the following information with respect to a bond:

(i) par value: 1000

(ii) term to maturity: 3 years

(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

The bond is sold at a price equal to its value.

Calculate the annual effective yield rate for the bond i.

(A) 8.1%

(B) 8.3%

(C) 8.5%

(D) 8.7%

(E) 8.9%

35.

The current price of an annual coupon bond is 100. The yield to maturity is an annual effective rate of 8%. The derivative of the price of the bond with respect to the yield to maturity is -700.

Using the bond’s yield rate, calculate the Macaulay duration of the bond in years.

(A) 7.00

(B) 7.49

(C) 7.56

(D) 7.69

(E) 8.00

36.

A common stock pays a constant dividend at the end of each year into perpetuity.

Using an annual effective interest rate of 10%, calculate the Macaulay duration of the stock.

(A) 7 years

(B) 9 years

(C) 11 years

(D) 19 years

(E) 27 years

37.

A common stock pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year.

Using an annual effective interest rate of 5%, calculate the Macaulay duration of the stock in years.

(A) 27

(B) 35

(C) 44

(D) 52

(E) 58

38. – 44. deleted

45.

You are given the following information about an investment account:

(i) The value on January 1 is 10.

(ii) The value on July 1, prior to a deposit being made, is 12.

(iii) On July 1, a deposit of X is made.

(iv) The value on December 31 is X.

Over the year, the time-weighted return is 0%, and the dollar-weighted (money-weighted) return is Y.

Calculate Y.

(A) -25%

(B) -10%

(C) 0%

(D) 10%

(E) 25%

46.

Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.

Calculate the principal repaid in the first payment.

(A) 444

(B) 454

(C) 464

(D) 474

(E) 484

47.

Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The price assumes an annual nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of i.

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate i.

(A) 9.50%

(B) 9.75%

(C) 10.00%

(D) 10.25%

(E) 10.50%

48.

A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of X to a fund for 25 years. The fund earns an annual nominal interest rate of 8% compounded monthly.

On his 65^{th} birthday, each 1000 of the fund will provide 9.65 of income at the beginning of each month starting immediately and continuing as long as he survives.

Calculate X.

(A) 324.70

(B) 326.90

(C) 328.10

(D) 355.50

(E) 450.70

49.

Happy and financially astute parents decide at the birth of their daughter that they will need to provide 50,000 at each of their daughter's 18^{th}, 19^{th}, 20^{th} and 21^{st} birthdays to fund her college education. They plan to contribute *X* at each of their daughter's 1^{st} through 17^{th} birthdays to fund the four 50,000 withdrawals. They anticipate earning a constant 5% annual effective interest rate on their contributions.

Let $v = 1/1.05$.

Determine which of the following equations of value can be used to calculate *X*.

(A) $X\sum_{k=1}^{17} v^k = 50,000[v + v^2 + v^3 + v^4]$

(B) $X\sum_{k=1}^{16} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(C) $X\sum_{k=0}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(D) $X\sum_{k=1}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(E) $X\sum_{k=0}^{17} v^k = 50,000[v^{18} + v^{19} + v^{20} + v^{21} + v^{22}]$

50. Delete

51.

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

Bond I: A 6-month bond with face amount of 1,000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually;

Bond II: A one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

Calculate the amount of each bond that Joe should purchase to exactly match the liabilities.

(A) Bond I – 1, Bond II – 0.97561

(B) Bond I – 0.93809, Bond II – 1

(C) Bond I – 0.97561, Bond II – 0.94293

(D) Bond I – 0.93809, Bond II – 0.97561

(E) Bond I – 0.98345, Bond II – 0.97561

52.

Joe must pay liabilities of 2000 due one year from now and another 1000 due two years from now. He exactly matches his liabilities with the following two investments:

Mortgage I: A one year mortgage in which X is lent. It is repaid with a single payment at time one. The annual effective interest rate is 6%.

Mortgage II: A two-year mortgage in which Y is lent. It is repaid with two equal annual payments. The annual effective interest rate is 7%.

Calculate $X + Y$.

(A) 2600

(B) 2682

(C) 2751

(D) 2825

(E) 3000

53.

Joe must pay liabilities of 1,000 due one year from now and another 2,000 due three years from now. There are two available investments:

Bond I: A one-year zero-coupon bond that matures for 1000. The yield rate is 6% per year

Bond II: A two-year zero-coupon bond with face amount of 1,000. The yield rate is 7% per year.

At the present time the one-year forward rate for an investment made two years from now is 6.5%

Joe plans to buy amounts of each bond. He plans to reinvest the proceeds from Bond II in a one-year zero-coupon bond. Assuming the reinvestment earns the forward rate, calculate the total purchase price of Bond I and Bond II where the amounts are selected to exactly match the liabilities.

(A) 2584

(B) 2697

(C) 2801

(D) 2907

(E) 3000

54.

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value *X* on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

Calculate *X*.

(A) 1400

(B) 1420

(C) 1440

(D) 1460

(E) 1480

55.

Toby purchased a 20-year par value bond with semiannual coupons of 40 and a redemption value of 1100. The bond can be called at 1200 on any coupon date prior to maturity, starting at the end of year 15.

Calculate the maximum price of the bond to guarantee that Toby will earn an annual nominal interest rate of at least 6% convertible semiannually.

(A) 1251

(B) 1262

(C) 1278

(D) 1286

(E) 1295

56.

Sue purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at par value X on any coupon date starting at the end of year 5. The lowest yield rate that Sue can possibly receive is an annual nominal rate of 6% convertible semiannually.

Calculate X.

(A) 1120

(B) 1140

(C) 1160

(D) 1180

(E) 1200

57.

Mary purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at 100 over the par value of 1100 on any coupon date starting at the end of year 5 and ending six months prior to maturity.

Calculate the minimum yield that Mary could receive, expressed as an annual nominal rate of interest convertible semiannually.

(A) 4.7%

(B) 4.9%

(C) 5.1%

(D) 5.3%

(E) 5.5%

58. Moved to Derivatives Section

59.

A liability consists of a series of 15 annual payments of 35,000 with the first payment to be made one year from now.

The assets available to immunize this liability are five-year and ten-year zero-coupon bonds.

The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio.

Calculate the amount invested in the five-year zero-coupon bonds.

(A) 127,000

(B) 167,800

(C) 208,600

(D) 247,900

(E) 292,800

60.

You are given the following information about a loan of L that is to be repaid with a series of 16 annual payments:

(i) The first payment of 2000 is due one year from now.

(ii) The next seven payments are each 3% larger than the preceding payment.

(iii) From the 9^{th} to the 16^{th} payment, each payment will be 3% less than the preceding payment.

(iv) The loan has an annual effective interest rate of 7%.

Calculate L.

(A) 20,689

(B) 20,716

(C) 20,775

(D) 21,147

(E) 22,137

61.

The annual force of interest credited to a savings account is defined by

$$\delta_t = \frac{\frac{t^2}{100}}{3 + \frac{t^3}{150}}$$

with t in years. Austin deposits 500 into this account at time 0.

Calculate the time in years it will take for the fund to be worth 2000.

(A) 6.7

(B) 8.8

(C) 14.2

(D) 16.5

(E) 18.9

62.

A 40-year bond is purchased at a discount. The bond pays annual coupons. The amount for accumulation of discount in the 15th coupon is 194.82. The amount for accumulation of discount in the 20th coupon is 306.69.

Calculate the amount of discount in the purchase price of this bond.

(A) 13,635

(B) 13,834

(C) 16,098

(D) 19,301

(E) 21,135

63.

Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68.

Calculate the total amount of interest paid on this loan.

(A) 1239

(B) 1647

(C) 1820

(D) 2319

(E) 2924

64.

Turner buys a new car and finances it with a loan of 22,000. He will make n monthly payments of 450.30 starting in one month. He will make one larger payment in n+1 months to pay off the loan. Payments are calculated using an annual nominal interest rate of 8.4%, convertible monthly. Immediately after the 18th payment he refinances the loan to pay off the remaining balance with 24 monthly payments starting one month later. This refinanced loan uses an annual nominal interest rate of 4.8%, convertible monthly.

Calculate the amount of the new monthly payment.

(A) 668

(B) 693

(C) 702

(D) 715

(E) 742

65.

Kylie bought a 7-year, 5000 par value bond with an annual coupon rate of 7.6% paid semiannually. She bought the bond with no premium or discount.

Calculate the Macaulay duration of this bond with respect to the yield rate on the bond.

(A) 5.16

(B) 5.35

(C) 5.56

(D) 5.77

(E) 5.99

66.

Krishna buys an n-year 1000 bond at par. The Macaulay duration is 7.959 years using an annual effective interest rate of 7.2%.

Calculate the estimated price of the bond, using duration, if the interest rate rises to 8.0%.

(A) 940.60

(B) 942.88

(C) 944.56

(D) 947.03

(E) 948.47

67.

The prices of zero-coupon bonds are:

Maturity	Price
1	0.95420
2	0.90703
3	0.85892

Calculate the third year, one-year forward rate.

(A) 0.048

(B) 0.050

(C) 0.052

(D) 0.054

(E) 0.056

68.

Sam buys an eight-year, 5000 par bond with an annual coupon rate of 5%, paid annually. The bond sells for 5000. Let d_1 be the Macaulay duration just before the first coupon is paid. Let d_2 be the Macaulay duration just after the first coupon is paid.

Calculate $\frac{d_1}{d_2}$.

(A) 0.91

(B) 0.93

(C) 0.95

(D) 0.97

(E) 1.00

69.

An insurance company must pay liabilities of 99 at the end of one year, 102 at the end of two years and 100 at the end of three years. The only investments available to the company are the following three bonds. Bond A and Bond C are annual coupon bonds. Bond B is a zero-coupon bond.

Bond	Maturity (in years)	Yield-to-Maturity (Annualized)	Coupon Rate
A	1	6%	7%
B	2	7%	0%
C	3	9%	5%

All three bonds have a par value of 100 and will be redeemed at par.

Calculate the number of units of Bond A that must be purchased to match the liabilities exactly.

(A) 0.8807

(B) 0.8901

(C) 0.8975

(D) 0.9524

(E) 0.9724

70.

Determine which of the following statements is false with respect to Redington immunization.

(A) Modified duration may change at different rates for each of the assets and liabilities as time goes by.

(B) Redington immunization requires infrequent rebalancing to keep modified duration of assets equal to modified duration of liabilities.

(C) This technique is designed to work only for small changes in the interest rate.

(D) The yield curve is assumed to be flat.

(E) The yield curve shifts in parallel when the interest rate changes.

71.

Aakash has a liability of 6000 due in four years. This liability will be met with payments of A in two years and B in six years. Aakash is employing a full immunization strategy using an annual effective interest rate of 5%.

Calculate $|A-B|$.

(A) 0

(B) 146

(C) 293

(D) 586

(E) 881

72.

Jia Wen has a liability of 12,000 due in eight years. This liability will be met with payments of 5000 in five years and B in $8+b$ years. Jia Wen is employing a full immunization strategy using an annual effective interest rate of 3%.

Calculate $\dfrac{B}{b}$.

(A) 2807

(B) 2873

(C) 2902

(D) 2976

(E) 3019

73.

Trevor has assets at time 2 of A and at time 9 of B. He has a liability of 95,000 at time 5. Trevor has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%.

Calculate $\dfrac{A}{B}$.

(A) 0.7307

(B) 0.9670

(C) 1.0000

(D) 1.0132

(E) 1.3686

74.

You are given the following information about two bonds, Bond A and Bond B:

i) Each bond is a 10-year bond with semiannual coupons redeemable at its par value of 10,000, and is bought to yield an annual nominal interest rate of i, convertible semiannually.

ii) Bond A has an annual coupon rate of (i + 0.04), paid semiannually.

iii) Bond B has an annual coupon rate of (i – 0.04), paid semiannually.

iv) The price of Bond A is 5,341.12 greater than the price of Bond B.

Calculate i.

(A) 0.042

(B) 0.043

(C) 0.081

(D) 0.084

(E) 0.086

75.

A borrower takes out a 15-year loan for 400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly.

Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now 409.88 lower than each payment from the original loan.

Calculate j.

(A) 4.72%

(B) 5.75%

(C) 6.35 %

(D) 6.90%

(E) 9.14%

76.

Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

The first bond has an annual nominal yield rate of 5% compounded semiannually, and a redemption value of 1200.

The second bond has an annual nominal yield rate of j compounded semiannually, and a redemption value of 800.

Calculate j.

(A) 2.20%

(B) 2.34%

(C) 3.53%

(D) 4.40%

(E) 4.69%