Fall 2012

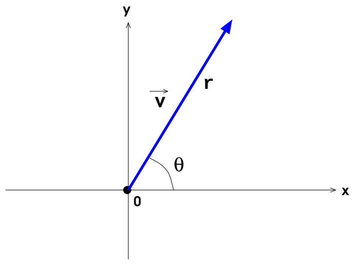
MA 16200

Study Guide - Exam # 1

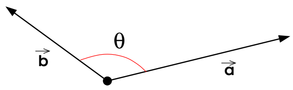
(1) Distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$; equation of a sphere with center (h, k, l) and radius r: $(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$.

(2) Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$; displacement vectors $\overrightarrow{PQ}$; vector arithmetic; components; Standard basis vectors $\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$, hence $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle = a_1\vec{\mathbf{i}} + a_2\vec{\mathbf{j}} + a_3\vec{\mathbf{k}}$; length (magnitude) of a vector $|\vec{\mathbf{a}}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; dot (or inner) product of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$: $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1b_1 + a_2b_2 + a_3b_3$; properties of dot products.

(3) Useful Vector: $\vec{\mathbf{v}} = (r\cos\theta)\,\vec{\mathbf{i}} + (r\sin\theta)\,\vec{\mathbf{j}}$:



(4) Angle between vectors: $\boxed{\cos\theta = \dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|}}$:

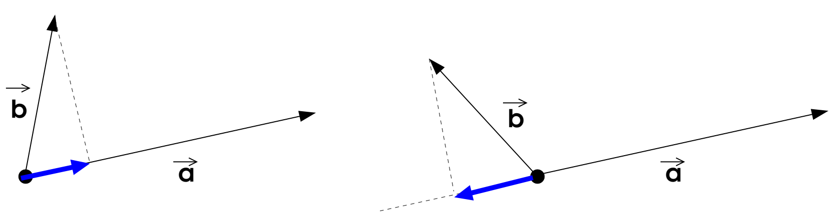


Perpendicular (orthogonal) vectors; if $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle$, then its direction cosines are $\cos\alpha = \dfrac{a_1}{|\vec{\mathbf{a}}|}$, $\cos\beta = \dfrac{a_2}{|\vec{\mathbf{a}}|}$, $\cos\gamma = \dfrac{a_3}{|\vec{\mathbf{a}}|}$, direction angles α, β, γ.

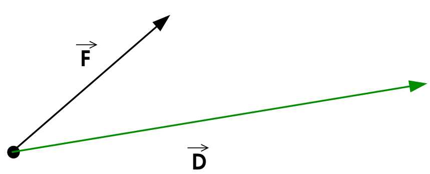
(5) Vector projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$: $\boxed{\text{proj}_{\vec{\mathbf{a}}}\vec{\mathbf{b}} = \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}\right)\dfrac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|}}$;

Scalar projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$: $\text{comp}_{\vec{\mathbf{a}}}\vec{\mathbf{b}} = \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}\right)$:

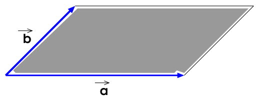
projection of b onto a

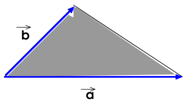


Work done by constant force is $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}}$:

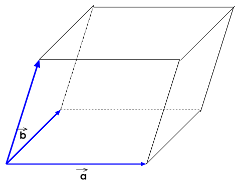


(6) Cross product: $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ (defined <u>**only**</u> for vectors in $\mathbb{R}^3$); $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \perp \vec{\mathbf{a}}$ and $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \perp \vec{\mathbf{b}}$; other properties of cross products; $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}| \sin\theta$;
$A = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| =$ area of parallelogram spanned by $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$;
$A = \frac{1}{2}|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| =$ area of triangle spanned by $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.



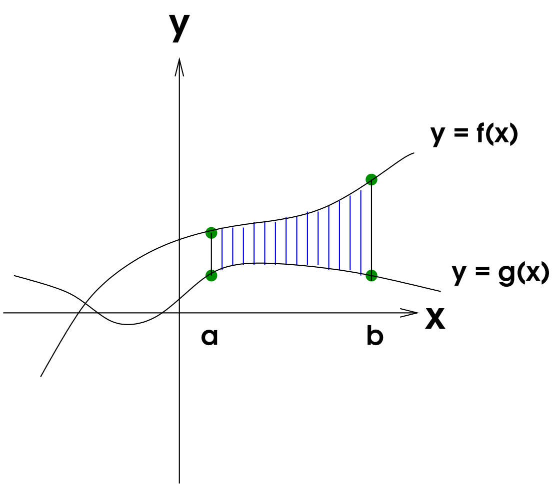


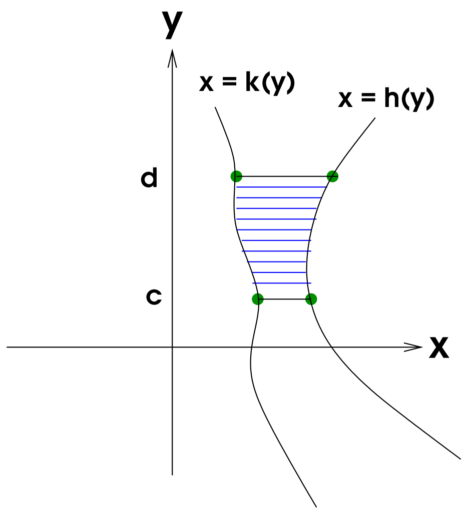
$V = |\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})| =$ volume of parallelopiped spanned by $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$:



(7) <u>APPLICATIONS OF INTEGRATION</u>

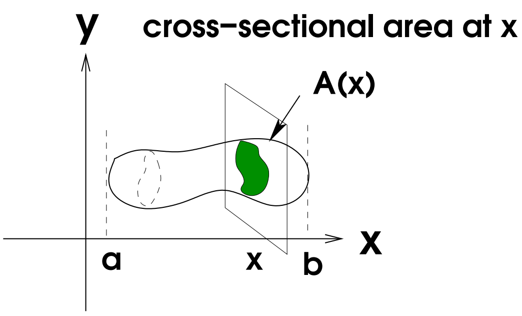
(a) <u>Areas Between Curves</u>: $A = \int_a^b \{f(x) - g(x)\}\,dx$ or $A = \int_c^d \{h(y) - k(y)\}\,dy$:

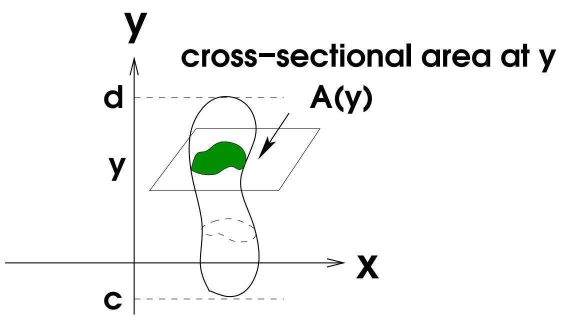




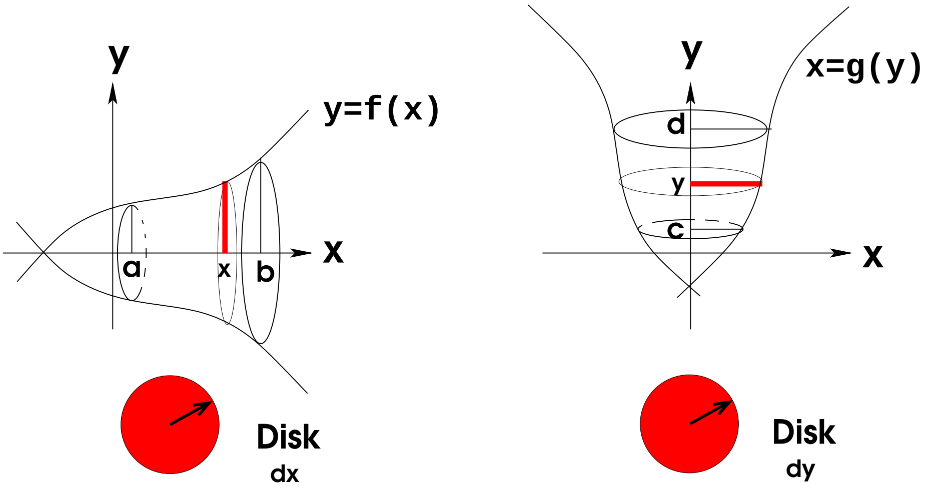
Note: If curves cross, you need to break up into several integrals.

(b) <u>Volumes of Solids by Cross-sectional Areas</u>: $V = \int_a^b A(x)\,dx$, or $V = \int_c^d A(y)\,dy$, where $A(x) =$ area of the cross-section of the solid with a plane $\perp$ x-axis at the point x, or $A(y) =$ area of the cross-section of the solid with a plane $\perp$ y-axis at the point y:

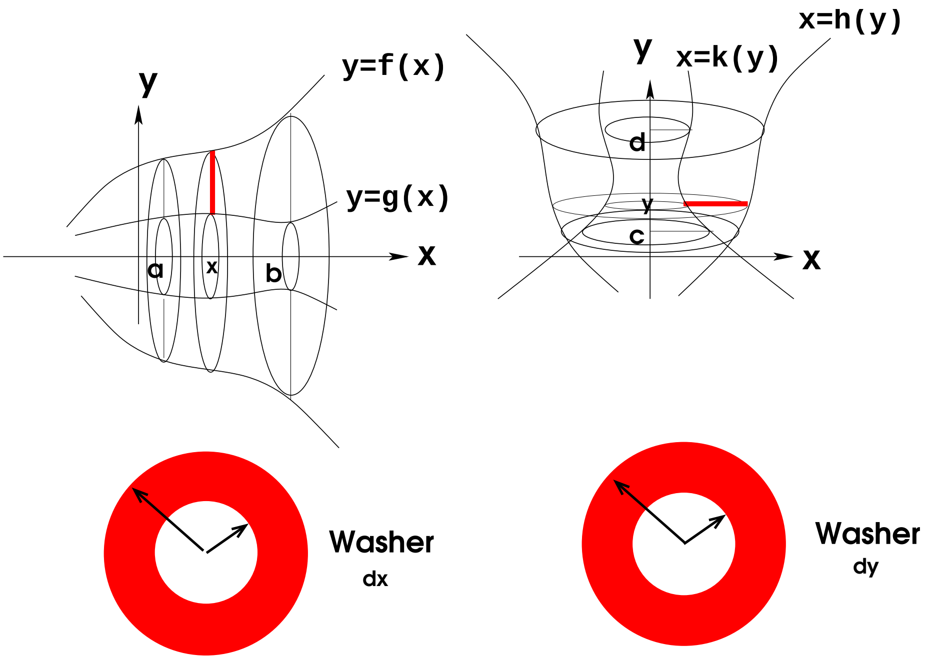




(c) <u>Volumes of Solids of Revolution by Disk/Washer Method</u>: Use Disk Method or Washer Method when you take cross-sectional areas *perpendicular* to axis of rotation. *In either case, the cross-section is always a disk/washer* :



$$\boxed{V = \int \pi R^2 \; \{\texttt{dx} \; or \; \texttt{dy}\}}$$



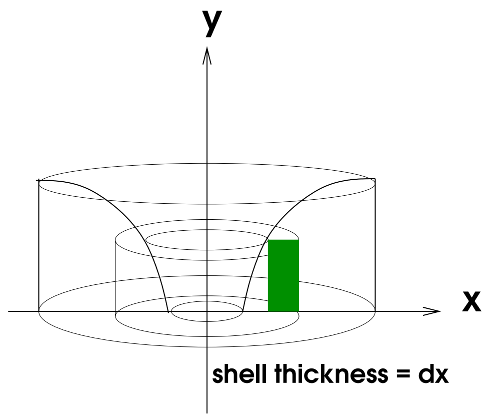
$$\boxed{V = \int \pi \left\{R^2 - r^2\right\} \; \{\texttt{dx} \; or \; \texttt{dy}\}}$$

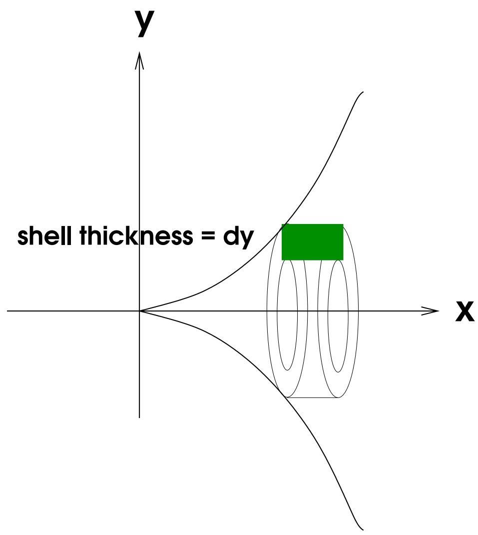
<u>**IMPORTANT**</u> - When to use dx or dy in Disk/Washer method?

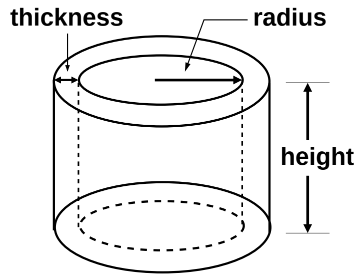
(i) If axis of rotation is $\parallel$ x - axis, use $\boxed{dx}$.

(ii) If axis of rotation is $\parallel$ y - axis, use $\boxed{dy}$.

(d) Volumes of Solids of Revolution by CYLINDRICAL SHELLS METHOD: Use Cyclindrical Shells Method when slabs of area are *parallel* to axis of rotation. Shell thickness is always either dx or dy.







$$V = \int 2\pi \left\{ \begin{array}{c}\texttt{shell}\\ \texttt{radius}\end{array} \right\} \left\{ \begin{array}{c}\texttt{shell}\\ \texttt{height}\end{array} \right\} \left\{ \begin{array}{c}\texttt{shell}\\ \texttt{thickness}\end{array} \right\} = \int 2\pi r h \,\{\texttt{dx}\ or\ \texttt{dy}\}$$

(e) If force F is constant and distance object moved along a line is d, then Work is $W = Fd$. Here are the English and Metric systems compared:

Quantity	*English System*	*Metric System*
Mass m	slug (= lb-sec^2/ft)	kilogram kg
Force F	pounds (lbs)	Newtons N (= kg-m/sec^2)
Distance d	feet	meters m
Work W	ft-lbs	Joules J (= kg-m^2/sec^2)
g	32 ft/sec^2	9.8 m/sec^2

If the force is variable, say $f(x)$, then Work $W = \int_a^b f(x)\,dx$; Hooke's Law: $f_s(x) = kx$; work done compressing/stretching springs, emptying tanks, pulling up chains.

(f) Average of a function over the interval $[a, b]$: $\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$;

`Mean Value Thm for Integrals`: $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$, for some $a \le c \le b$.

(8) Techniques of Integration

(a) **Simple Substitution Method** : $\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$ (let $u = g(x)$)

(b) **Integration by Parts**: $\boxed{\int u\, dv = uv - \int v\, du}$; the **LIATE** rule:

let $u =$ **L**og **I**nv trig **A**lg **T**rig **E**xp.

(c) **Trig Integrals**: Integrals of the type $\int \sin^m x \cos^n x\, dx$ and $\int \tan^m x \sec^n x\, dx$

Some useful trig identities:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ and $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

(iii) $\sin 2\theta = 2\sin\theta\,\cos\theta$

(iv) $\tan^2\theta + 1 = \sec^2\theta$

Some useful trig integrals:

(i) $\int \tan u\; du = \ln|\sec u| + C$

(ii) $\int \sec u\; du = \ln|\sec u + \tan u| + C$

(d) **Trig integrals of the form**: $\int \sin mx\, \sin nx\; dx$, $\int \cos mx\, \cos nx\; dx$, $\int \sin mx\, \cos nx\; dx$,

use these trig identities:

$$\sin A\, \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$$

$$\cos A\, \cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$$

$$\sin A\, \cos B = \frac{1}{2}\{\sin(A-B) + \sin(A+B)\}$$

(e) **Trigonometric Substitutions**:

*Expression**	*Trig Substitution*	*Identity needed*
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

* Or powers of these expressions.