MA 351 final review problems

Version as of April 19th.

The final will be as scheduled here. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. All of the problems from both midterm reviews.

2. Let
$$A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ -1 & -2 & 1 & 5 \\ 3 & 6 & 2 & 0 \end{bmatrix}.$$
   (a) Find the rref, rank, nullity, a basis for the range, and a basis for the nullspace.

   (b) Find a basis for the range that has all integer entries and as many zero entries as possible.

   (c) Find an orthonormal basis for the nullspace.

3. Let $\vec{v}$ and $\vec{w}$ be vectors wuch that $\|\vec{v}\| = 2$, $\|2\vec{v} - 3\vec{w}\| = 6$, and $\vec{v} \cdot \vec{w} = 0$. Find $\|\vec{w}\|$.

4. More problems to be added soon.

Answer key

1.

2. (a) $\begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, 2, 2, $\left(\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \right)$, and $\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right)$.

   (b) $\left(\begin{bmatrix} 12 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix} \right)$.

   (c) $\left(\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{15\sqrt{30}} \begin{bmatrix} 2 \\ 4 \\ 15 \\ -5 \end{bmatrix} \right)$.

3. $\frac{2}{3}\sqrt{5}$.