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MA 366
Spring 2019

Homework 4

Due Feburary 12th at the beginning of class, or by 1:50 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Find the general solution to *Legendre's equation*:

$$(1-x^2)y''(x) - 2xy'(x) + n(n+1)y(x) = 0.$$

   when $n = 1$, for $x \in (-1, 1)$. Express your answer using one natural logarithm, one square root, and no absolute values.

   Hint: First find a solution of the form x^m for a suitable choice of m. Then use the reduction of order substitution $y(x) = u(x)x^m$. You may use the following partial fraction decompositions without verifying them:

$$\frac{4x^2-2}{x(x^2-1)} = \frac{1}{x-1} + \frac{2}{x} + \frac{1}{1+x}, \qquad \frac{1}{x^2(x^2-1)} = \frac{1}{2(x-1)} - \frac{1}{x^2} - \frac{1}{2(x+1)}.$$

2. Exercise 2.2.6 from page 72 of `https://www.jirka.org/diffyqs/diffyqs.pdf`

3. Solve

$$y'' - 3y' + 2y = 0, \qquad y(0) = 0, \qquad y'(0) = 1.$$

4. Let a be a given real number. Find all solutions to

$$y'' - 3y' + 2y = 0, \qquad y(0) = 0, \qquad y'(a) = 1.$$

   How does the number of solutions depend on a?

5. Let a and b be constants such that the differential equation

$$y'' + ay' + by = 0$$

   has

$$y(t) = C_1e^{2t} + C_2e^{-5t}$$

   as its general solution.

   (a) Find a and b.

   (b) Find c such that the solution to the initial value problem

$$y'' + ay' + by = 0, \qquad y(0) = 1, \qquad y'(0) = c,$$

   obeys $\lim_{t\to\infty} y(t) = 0$.