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MA 510
Spring 2020

Homework 3

Due February 5th at the beginning of class, or by 12:30 pm in MATH 602. Justify your answers. Please let me know if you have a question or find a mistake.

1. Let $f(x,y) = \sin(x+y^2)$. Find the linear approximation to f at $(0, \sqrt{\pi/4})$

2. Let $f(x,y,z) = \sqrt{x^2+y^2+z^2}$. Without using a calculator, find the linear approximation to f at $(1320231, -2640462, 3960693)$.

3. A terrain of hills has altitude in meters at a horizontal point (x,y) given by $f(x,y) = x^2+y^2-x^4-y^6$. The horizontal position at time t of a train traveling these hills is given by $g(t) = (\cos t, \frac{1}{2}\sin t)$. In meters per second, what is the instantaneous rate of change of altitude of the train at time $t=0$?

4. (a) Let $z = \sin(x-y)$. Use the chain rule to evaluate
$$\partial_x z + \partial_y z.$$
   (b) Let $z = f(ax+by)$, where a and b are given constants, and f is a given differentiable function. Use the chain rule to find all constants c and d such that
$$c\partial_x z + d\partial_y z = 0.$$

5. Let $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ and $g\colon \mathbb{R}^3 \to \mathbb{R}^2$ be given by
$$f(x_1,x_2) = (e^{x_1+2x_2},\ x_1^2 - \cos x_2,\ x_1^2 + x_2 - 2)$$
$$g(y_1,y_2,y_3) = (y_1^2 + y_2^4 + \cos y_3,\ y_1 + y_2^2 + \sin y_3).$$
   (a) Let $F(y_1,y_2,y_3) = f(g(y_1,y_2,y_3))$. Find $DF(0,0,0)$.
   (b) Let $G(x_1,x_2) = g(f(x_1,x_2))$. Find $DG(0,0)$.

   Hint: It is not necessary to write out F or G.