MA 527
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Homework 11

Due December 7th in class or by 1:50 pm in MATH 602.

This homework covers sections 11.7, 11.8, 11.9, and 12.7.

1. In class we derived the formula
$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(w) e^{-w^2 t} e^{iwx} dw,$$
for the solution to
$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad u(x,0) = f(x).$$
Derive the corresponding formula for the solution to
$$\partial_t u(x,t) = \partial_x^3 u(x,t), \qquad u(x,0) = f(x).$$

2. (a) Let $a > 0$. Find the Fourier transform of
$$f(x) = e^{-a|x|}.$$
(b) Let $b > 0$. Use your answer to part (a) to find the Fourier transform of
$$f(x) = \frac{1}{x^2 + b}.$$

3. Another version of the formula for the solution to
$$\partial_t u(x,t) = \partial_x^2 u(x,t), \qquad u(x,0) = f(x),$$
is
$$u(x,t) = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{\infty} f(y) e^{-\frac{(x-y)^2}{4t}} dy.$$
Use this version to find constants $A > 0$ and $B > 0$ such that
$$u(x, 1/4) = Ae^{-Bx^2},$$

when $f(x) = e^{-x^2}$.

Hint: Use completion of squares to Write the integrand as $e^{-a(y+bx)^2+cx^2}$ for suitable constants a, b, and c, and then use the substitution $v = y + bx$, $dv = dy$, and then use $\int_{-\infty}^{\infty} e^{-av^2} dv = \sqrt{\pi/a}$.

Aside: The same method will find $u(x, t)$ for more general Gaussian inital conditions and more general times $t > 0$, but with a more complicated formula. (You can ignore this issue for this homework).