Homework 2

Due September 7th in class or by 1:50 pm in MATH 602.

1. (a) Find a basis for the nullspace of the matrix

$$\begin{bmatrix} 1 & -1 & 2 & -2 \\ 2 & 1 & 2 & 1 \\ -1 & -2 & 0 & -3 \end{bmatrix}$$

   (b) Give a *single* solution to the system

$$x_1 - x_2 + 2x_3 - 2x_4 = 1$$
$$2x_1 + x_2 + 2x_3 + x_4 = 2$$
$$-x_1 - 2x_2 - 3x_4 = -1$$

   Hint: This should not require any calculations.

   (c) Use your answers to parts (a) and (b) give *all* the solutions to the system in part (b).

2. Compute the determinants of the following matrices.

   Hint: It may be helpful to use row operations.

   (a) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & -1 \\ -3 & 4 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & -5 & 0 & 0 \\ -3 & 4 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

(c)
$$\begin{bmatrix} -2 & 2 & 0 & 0 & 1 \\ 0 & 4 & 0 & 0 & 2 \\ 1 & -2 & 3 & 0 & 0 \\ 2 & -3 & 4 & -5 & 0 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

3. Let A be a 3x3 matrix such that
$$A^4 = 2A^3.$$

(a) What are the possible values of $\det A$?

(b) If it is known additionally that A is invertible, what is A?

4. For which values of c is the following matrix invertible? Compute the inverse for those values.
$$\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 4 \\ 2 & 1 & c \end{bmatrix}$$