MA 527
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Fall 2017

Homework 7

Due October 26th in class or by 1:50 pm in MATH 602.

This homework covers sections 12.12, 11.1, and 11.2.

1. Use the Laplace transform to find the function $w(x,t)$ such that

$$\begin{aligned} \partial_x w(x,t) &= -2\partial_t w(x,t) + 3w(x,t), && \text{when } x>0 \text{ and } t>0,\\ w(x,0) &= \partial_t w(x,0) = 0, && \text{when } x \geq 0,\\ w(0,t) &= t^2 e^{-t}, && \text{when } t \geq 0. \end{aligned}$$

2. Let $f(x) = x$ for $0 < x < 1$.

   (a) Let f_1 be the extension of f with period 1. Sketch f_1 and find its Fourier series.

   (b) Let f_2 be the even extension of f with period 2. Sketch f_2 and find its Fourier series.

   (c) Let f_3 be the odd extension of f with period 2. Sketch f_3 and find its Fourier series.

3. Let $f(x)$ have the Fourier series

$$2 + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x,$$

and let $g(x) = 2f(2x - \frac{1}{2})$. Use the formulas expanding $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$ to write out the partial sum of the Fourier series of $g(x)$ up to the terms $\sin(8\pi x)$ and $\cos(8\pi x)$.