MA 527
Kiril Datchev
Fall 2017

Homework 8

Due November 2nd in class or by 1:50 pm in MATH 602.

This homework covers sections 11.3, 11.4, 11.5, and 11.6.

1. (a) Given $n > 0$, find constants A_n and B_n such that

$$y_n(t) = A_n \cos nt + B_n \sin nt$$

is a solution to

$$y_n''(t) + .01y_n'(t) + 4y_n(t) = \cos nt.$$

(b) With $y_n(t)$ as in part (a), find a constant $C_n > 0$ such that

$$y_n(t) = C_n \cos(nt - \varphi_n).$$

(You do not need to find φ_n).

(c) Use your answer to part (a) to find a periodic solution $y_p(t)$ to

$$y_p''(t) + .01y_p'(t) + 4y_p(t) = f(t),$$

where

$$f(t) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \cos nt,$$

and find the Fourier series of y_p.

(d) Use your answer to part (b) to find constants $D_n > 0$ such that

$$y_p(t) = \sum_{n=1}^{\infty} D_n \cos(nt - \varphi_n).$$

(You do not need to find φ_n). For which value of n is D_n largest?

2. Let $f(x) = |x|$. Find constants a_0, a_1, and b_1 such that the error

$$E = \int_{-\pi}^{\pi} (f(x) - g(x))^2 dx$$

is minimized, where

$$g(x) = a_0 + a_1 \cos x + b_1 \sin x.$$

Also evaluate E and sketch a graph of f and g on the interval $[-\pi, \pi]$.

3. Find the eigenvalues and eigenfunctions of

$$y'' + 2y' + (\lambda + 2)y = 0, \qquad y(0) = y(1) = 0.$$

With respect to which inner product on the interval $[0, 1]$ are the eigenfunctions orthogonal?