Math 182 Recitation3-6

Due at recitation, Thurs. Mar. 6, 2008

1. (a) Show that the gravitational potential of a mass distributed uniformly throughout a spherical shell $a \leq \rho \leq b$ (spherical coordinates), where $0 < a < b$, is constant in the interior of the shell.

(b) What gravitational force does the mass exert on a particle in the interior of the shell?

2. In this problem you will calculate (hopefully) the potential produced by a mass distributed uniformly along a line segment—which you can think of as an extremely narrow cylinder.

(a) Work first in the (x, y)-plane. Choose coordinates so that the line segment goes from $(-1, 0)$ to $(1, 0)$. Choose units so that the density function is $\delta(\xi) \equiv 1$. Using formulas in the boxes on pages 530 and 531 of the text, show that the potential at a point (x, y) off the segment is

$$\ln\left(\frac{\sqrt{(x-1)^2+y^2}-(x-1)}{\sqrt{(x+1)^2+y^2}-(x+1)}\right).$$

(b) Using the formula $\coth a = (e^{2a}+1)/(e^{2a}-1)$, prove that

$$\coth\left(\frac{1}{2}\ln\left(\frac{b}{c}\right)\right) = \frac{b+c}{b-c}.$$

(c) Deduce from (b) that the potential in (a) can be written as

$$2\coth^{-1}\left(\frac{r_1+r_2}{2}\right)$$

where r_1 and r_2 are the distances of (x, y) from the two end points of the segment.

(d) Now think of the (x, y)-plane as sitting in $\mathbf{R}^3$. Show that the equipotential surfaces (level surfaces of the potential function) are ellipsoids obtained by rotating around the x-axis ellipses in the (x, y)-plane with foci at the end points of the segment.

(e) Why is this potential NOT the same as the potential of the entire mass concentrated at the centroid of the segment? (Hint: Use (d).)