Eddie Price Solving Systems of Equations with MATLAB Summer 2016

This document is a guide to solving systems of equations using MATLAB. If you have taken linear algebra, you should already know this. Solving systems of equations can be quite tedious by hand, particularly if you have 4 or more unknowns. Luckily, matrices give us a more organized approach to solving systems of linear equations.

The first step is to organize your system of equations in a coherent manner. Since we will often be using systems of equations to solve initial value problems, usually our "variables" will be of the form $c_1, c_2, ..., c_n$. Arranging your equations so that c_1 is the first "variable," c_2 is the second "variable," and so forth is the best option to go. We will also be using partial fraction decomposition and the Method of Undetermined Coefficients, where we have A, B, C, etc., and usually alphabetical order will be best. You should get the constant by itself on the other side of the equals sign, on the right. In order words, your system of equations should look something like this:

$$\begin{aligned} c_1 + 2c_2 \qquad\quad - c_4 &= -1 \\ -c_1 \qquad\quad + c_3 + 2c_4 &= 4 \\ 5c_1 \; + c_2 \; + c_3 - 3c_4 &= 10 \\ 4c_2 + 5c_3 - 6c_4 &= 10 \end{aligned}$$

We then form a 4×5 matrix using the *coefficients* of the variables. The first row will be the first equation, second row will be the second equation, etc. The first column will correspond to the coefficient of the first variable, second column will correspond to the coefficient of the second variable, etc. The last column will have the constant after the equal sign. The matrix will look like this:

$$\begin{bmatrix} 1 & 2 & 0 & -1 & -1 \\ -1 & 0 & 1 & 2 & 4 \\ 5 & 1 & 1 & -3 & 10 \\ 0 & 4 & 5 & -6 & 10 \end{bmatrix}$$

We then use the "rref" command in MATLAB. "rref" tells MATLAB to put this matrix into what is called "Reduced Row Echelon Form." A matrix is put into RREF by using elementary row operations. These operations include interchanging two rows, multiplying one row with a nonzero constant, and adding a constant multiple of one row to another row. These are the types of operations we do using the elimination technique for solving systems of linear equations.

To enter this matrix into MATLAB, we enter the following command:

A=[1 2 0 -1 -1; -1 0 1 2 4; 5 1 1 -3 10; 0 4 5 -6 10]

The above command names the matrix "A" (you can choose to name it something else if you prefer). You enter all entries of the matrix, row by row, leaving spaces between each entry, and separating each row by a semicolon.

Next, enter the following command:

rref(A)

This command takes the reduced row echelon form of your above matrix (named "A"). You should get a matrix of the form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

This gives you a new system of equations in which the values of the variables are clear. In particular, this tells you

$$c_1 = 2,\ \ c_2 = -1,\ \ c_3 = 4,\ \ c_4 = 1$$