Eddie Price **Special Shifts of Trig Functions** **Summer 2016**

Here are the shifts you are expected to know, where n is any integer:

$$\sin(t + (2n+1)\,\pi) = -\sin t, \qquad \cos(t + (2n+1)\,\pi) = -\cos t$$
$$\sin(t + 2n\pi) = \sin t, \qquad \cos(t + 2n\pi) = \cos t$$

The first line is saying that if you add any *odd* multiple of π on the inside of your trig function, you negate the trig function, and the second line is saying that if you add any *even* multiple of π on the inside of your trig function, you have done nothing to it.

Since the trig functions are periodic with period 2π, it's not difficult to see why the second line is true: you're shifting the function n whole periods, so you have not change the output at all.

Notice that for the first line, $(2\,(n+1)+1)\,\pi - (2n+1)\,\pi = 2\pi$, using what we know from the second line, we see that shifting any odd multiple of π is the same as shifting by π.

When looking at the unit circle, shifting the function by π is the same thing as adding π to the angle. This is the same thing as rotating the unit circle one half-turn, which is the same thing as reflecting the unit circle through the origin. Given a point $P = (x, y)$ on the Cartesian plane, the reflection of P through the origin is the point $(-x, -y)$.

Recall that given an angle θ, the corresponding point on the unit circle is $(\cos\theta, \sin\theta)$, so its reflection is $(-\cos\theta, -\sin\theta)$.

In other words, for any real number t, $(\cos(t+\pi)\,, \sin(t+\pi)) = (-\cos t, -\sin t)$. So $\cos(t+\pi) = -\cos t$ and $\sin(t+\pi) = -\sin t$.

In more concrete form, the equations can be written as:

$$-\sin t = \sin(t \pm \pi) = \sin(t \pm 3\pi) = \sin(t \pm 5\pi) = \dots$$
$$-\cos t = \cos(t \pm \pi) = \cos(t \pm 3\pi) = \cos(t \pm 5\pi) = \dots$$
$$\sin t = \sin(t \pm 2\pi) = \sin(t \pm 4\pi) = \sin(t \pm 6\pi) = \dots$$
$$\cos t = \cos(t \pm 2\pi) = \cos(t \pm 4\pi) = \cos(t \pm 6\pi) = \dots$$