

Fiber & cofiber sequences.

Given $f: X \rightarrow Y$, consider

$$\text{hocolin} \left(\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \downarrow & \lrcorner & \downarrow \\ * & \longrightarrow & \end{array} \right) = Z$$

Z is the cofiber of f .

ex. If X, Y are cofibrant,

$$\begin{array}{ccc} & \xrightarrow{CX} & \xrightarrow{\sim} \\ X & \xrightarrow{\quad} & * \end{array}$$

Then $X \rightarrow Y$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ CX & \longrightarrow & CX \amalg_X Y = \text{cof}(X \rightarrow Y) \end{array}$$

Likewise,

$$\begin{array}{ccc} \text{fib}(f) & \rightarrow & * \\ \downarrow & \lrcorner & \downarrow \\ X & \xrightarrow{f} & Y \end{array}$$

If X, Y are fibrant, then

$$\begin{array}{ccc} * & \xrightarrow{\quad} & Y \\ \downarrow & \lrcorner & \downarrow \\ X & \xrightarrow{f} & Y \end{array}$$

PY .

$$\begin{array}{ccc} X \times_Y PY = \text{fib} & \longrightarrow & PY \\ \downarrow & \lrcorner & \downarrow \\ X & \longrightarrow & Y \end{array}$$

$$\text{ex. } \begin{array}{ccc} X & \longrightarrow & * \\ \downarrow & \lrcorner & \downarrow \\ * & \longrightarrow & \Sigma X \end{array} \qquad \begin{array}{ccc} \Omega X & \longrightarrow & * \\ \downarrow & \lrcorner & \downarrow \\ * & \longrightarrow & X \end{array}$$

$$\begin{array}{ccc} X & \longrightarrow & X \vee Y \\ \downarrow & \lrcorner & \downarrow \\ * & \longrightarrow & Y \end{array} \qquad \begin{array}{ccc} Y & \longrightarrow & X \times Y \\ \downarrow & \lrcorner & \downarrow \\ * & \longrightarrow & X \end{array}$$

Def. A pointed model category is one with a zero object.

A cofiber sequence is:
 $X \xrightarrow{f} Y \longrightarrow Z$
" $\text{cof}(X \rightarrow Y)$.

Observation, $\begin{array}{ccccc} X & \longrightarrow & Y & \longrightarrow & * \\ \downarrow & \lrcorner & \downarrow & \lrcorner & \downarrow \\ * & \longrightarrow & Z & \longrightarrow & \Sigma X \end{array}$

$$X \longrightarrow Y \longrightarrow Z \longrightarrow \Sigma X.$$

Likewise, if $F \rightarrow X \rightarrow Y$ is a fiber sequence,