STAT 472 Midterm Fall Study Sheet Fall 2007

Assume uniform distribution of deaths over each year (UDD) whenever necessary to answer the question. You must indicate each time UDD is used! Be careful: Saying that you need UDD when you don't will be penalized.

Notation: $T(x)$ is the time until death at age (x), $K(x) = [T(x)]$ is the years until death at age (x).

In order to limit the scope of the test slightly, I do not plan to ask questions on increasing or decreasing insurance and annuities. I also don't plan to ask you to compute distribution functions or percentiles.

(1) Translate into insurance language: A_{25}, $\overline{A}_{25}$, $A^{1}_{25:\overline{10}|}$, $\overline{A}^{1}_{25:\overline{10}|}$, $A_{25:\overline{10}|}$, $\overline{A}_{25:\overline{10}|}$, a_{25}, $\overline{a}_{25}$, $\ddot{a}_{25}$, $\ddot{a}_{25:\overline{10}|}$, ${}_{10}E_{25}$, $\mathring{e}_{25}$, e_{25}.

Sample answers: "A_{25} is the actuarial present value of a \$1 whole life insurance policy payable at the end of the year of death issued at age 25." and "$\overline{a}_{25:\overline{10}|}$ is the actuarial present value of a 10 year temporary annuity issued at age 25 that pays 1 continuously."

Remark. In class I incorrectly said that $a_x = \nu \ddot{a}_x$. I corrected this to

$$\begin{aligned} a_x &= \sum_{k=0}^{\infty} \nu^{k+1}\ {}_{k+1}p_x \\ &= \sum_{k=1}^{\infty} \nu^{k}\ {}_{k}p_x \end{aligned}$$

The latter sum is the formula for $\ddot{a}_x$, minus its first term. Hence

$$a_x = \ddot{a}_x - 1$$

This makes sense: the only difference between a_x and $\ddot{a}_x$ lies in whether or not we receive the first payment.

(2) Below is a life table for male, non-smoking, jubb-jub birds. Write an *explicit* expression that computes the following quantities for members of this group, assuming that $\delta = .05$. **Do not evaluate the expression.** Thus, for example if the question were to compute ${}_2E_1 + {}_1E_1$, the answer would be

"$e^{-2(.05)}\frac{1}{50}+e^{-.05}\frac{10}{50}$".

(x)	l_x
0	100
1	50
2	10
3	1
4	0

(a) The single payment pure premium for a \$5,000 whole life policy issued at age 1 payable at (i) the end of the year of death and (ii) at the time of death.
(b) The single payment pure premium for an insurance policy issued at birth, payable at the end of the year of death, which pays \$1 if death occurs in the first two years, \$0 if it in the third year, and \$2 if it occurs occurs any time thereafter.
(c) $Var(Z)$ where $Z=\nu^{K(2)+1}$.
(d) $Var(Z)$ where $Z=\nu^{T(2)}$.
(e) $Var(W)$ where $W(n)=n^2, n=0,1,2,3,4$.
(f) The actuarial present value of a 2 year temporary annuity paid at the beginning of the year, issued to a new born jubb-jub bird.
(g) The actuarial present value of a continuous 2 year temporary annuity issued to a new born jubb-jub bird.
(h) The expected remaining years of life (curtate expectancy of life) of a 1 year old.

(3) Given that $\delta = .08$ and $\mu(x) = 3x^2$, find an explicit expression for the following quantities. **You may leave all integrals, except the integral of μ, unevaluated.**
(a) $s(x)$ (the survival function)
(b) The single payment pure premium for a 10 year term insurance policy issued at age 30 paying 1 at the time of death,
(c) the complete expectation of life of an individual aged 30,
(d) the actuarial present value of a continuous whole life annuity paying 1 issued at age 30,
(e) $Var(T(30))$,
(f) $E(T(30)^3)$ (E is "expected value").

(g) $P(T(30) \leq 20)$.

(4) Repeat Problem 3, assuming that $l_x = e^{-x^2}$ instead of $\mu(x) = 3x^2$. Also find a formula for μ_x.

(5) There is an "Illustrative Life Table" posted on the class web page. (This table will be provided on the test.) Use this table to find (assuming that $i = 0.06$)
   (a) $Var(Z)$ where $Z = \nu^{K(40)+1}$
   (b) $Var(Z)$ where $Z = \nu^{T(40)}$
   (c) The actuarial present value of an annuity issued at age 26 that pays \$200 per year continuously.

(6) We have issued \$1,000 whole life policies, payable at the time of death, to each of 200 identically distributed, independent lives at age 27. Assume that $\delta = .02$, $\mu = .03$. Use the normal approximation to determine the size of the fund necessary to have on hand in order to be 90% certain of being able to pay any claim. *The normal distribution table posted on the course web page will be provided on the test. This is the same table provided on the MLC exam.*

(7) There is an "Illustrative Life Table" posted on the class web page. (This table will be provided on the test.) Use this table to compute the single payment pure premium of a 40 year \$2,000 term policy, payable at the end of the year of death, issued to a 20 year old where $i = .06$. *Hint:*

$$A_x = A^1_{x:\overline{m}|} + \nu^m\, {}_mp_x A_{x+m}$$

This hint will NOT be provided on the test!