

MATH 341, ASSIGNMENT #4
DUE: WEDNESDAY, FEBRUARY 13, IN CLASS

Q1. Exercise 4.2.1 on p. 58.

Q2. Show that if $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| < 1$, then $\lim_{n \rightarrow \infty} a_n = 0$.

Q3. Exercise 5.3.3 on p. 74.

Q4. Exercise 5.4.1 on p. 74.

Q5. Show that $\lim_{n \rightarrow \infty} \sum_{i=n}^{2n} 1/i = \ln(2)$. (As a warm-up, can you show the limit, if it exists, must be between $1/2$ and 1 ?) Use the method from Example 5.2C on p. 66

Q6. A modification of Example 5.2C in the book shows that $\lim_{n \rightarrow \infty} \frac{\ln(n!)}{n \ln(n)} = 1$. Stirling's formula shows that

$$n! \leq e\sqrt{n}(n/e)^n.$$

Is this limit sufficient to derive Stirling's formula? If not, what estimate of the error term is needed?

Q7. Exercise 6.1.2 on p. 89.

Q8. Exercise 6.4.2 on p. 90.

Q9. Exercise 6.5.3, only parts (a)(c)(e)(g), on p. 90.

Q10. Given an example of a sequence whose set of cluster points is exactly $[0, 1]$. Is it possible to find a sequence whose cluster points are exactly the set $(0, 1)$? Find an example or disprove.

Q11. Problem 6-6 on p. 92.

Challenge. Show the set of cluster points of $\sin(n)$ is exactly $[-1, 1]$.