MATH 341, QUIZ #2
MONDAY, JANUARY 28

Question 0.0.1. Let (a_n) be a sequence of integers. Show that $\lim_{n\to\infty} a_n = L$ if and only if L is an integer and (a_n) is eventually constant with value L.

Proof. ($\Rightarrow$) For $\epsilon = 1/2$ we have that there exists N so that for all $n \geqslant N$, $|a_n - L| < 1/2$. By the triangle inequality

$$|a_n - a_m| \leqslant |a_n - L| + |a_m - L| < 1/2 + 1/2 = 1.$$

Thus $a_n = a_m$ for all $n, m \geqslant N$ since distinct integers are at least distance 1 apart, so (a_n) is eventually constant with value a_N. From the definition of the limit and the fact that $a_n = a_N$ for all $n \geqslant N$, we have that $|a_N - L| < \epsilon$ for all $\epsilon > 0$, so $L = a_N$ is an integer.

($\Leftarrow$) There exists N so that for all $n \geqslant N$, $a_n = L$, thus for every $\epsilon > 0$ we have $|a_n - L| = 0 < \epsilon$ for all $n \geqslant N$, so the limit is seen to converge to L.

□