MATH 341, QUIZ #5
MONDAY, MARCH 25

Question 0.0.1. Show that $e^x < 1 + 3x$ for $x \approx 0$, $x > 0$, by using the Mean Value Theorem on the interval $[0, x]$.

Solution. e^x is continuous and differentiable on the entire real line, so it is continuous and differentiable on any interval. For any $x > 0$ consider the interval $[0, x]$. Since e^x is its own derivative, by the Mean Value Theorem, there is $c \in (0, x)$ so that

$$e^c = \frac{e^x - e^0}{x - 0}.$$

This shows that $e^x - 1 = xe^c$, or $e^x = 1 + e^c x$.

If $c < 1$, then $e^c < e^1 < 3$, so

$$e^x = 1 + e^c x < 1 + 3x$$

when c can be guaranteed to be less than 1. Since $c \in (0, x)$, this will happen if $x < 1$. Thus, $e^x < 1 + 3x$ if $x < 1$. □