MATH 341, QUIZ #6
WEDNESDAY, APRIL 3

Question 0.0.1. Using only the definition, show that the function x is Riemann integrable on $[0, 1]$.

Solution. Let $\mathcal{P} = \{0 = x_0 < x_1 < \cdots < x_n = 1\}$ be a partition of $[0, 1]$ with $|\mathcal{P}| < \delta$. Since x is increasing, we have that

$$U_x(\mathcal{P}) = \sum_{i=0}^{n-1} x_{i+1}(x_{i+1} - x_i)$$

and

$$L_x(\mathcal{P}) = \sum_{i=0}^{n-1} x_i(x_{i+1} - x_i).$$

Therefore, by combining like terms,

$$U_x(\mathcal{P}) - L_x(\mathcal{P}) = \sum_{i=0}^{n-1} (x_{i+1} - x_i)(x_{i+1} - x_i).$$

Since $|\mathcal{P}| < \delta$, $0 < x_{i+1} - x_i < \delta$ for all $i = 1, \dots, n$, so

$$\sum_{i=0}^{n-1} (x_{i+1} - x_i)(x_{i+1} - x_i) < \sum_{i=0}^{n-1} \delta(x_{i+1} - x_i) = \delta(1 - 0) = \delta.$$

Therefore, given any $\epsilon > 0$, by choosing $\delta = \epsilon$ we have that $U_x(\mathcal{P}) - L_x(\mathcal{P}) < \epsilon$ when $|\mathcal{P}| < \delta$. This verifies the definition of Riemann integrability for x. $\square$