MATH 490, WORKSHEET #1
WEDNESDAY, JANUARY 15, 2020

Problem 1, Larson. A set X has a binary operation $*$ satisfying the following two properties: (1) $x * x = x$; $(x * y) * z = (y * z) * x$ for all x, y, z in X. Show that $x * y = y * x$ for all x, y in X.

Problem 2, Larson. Find n and positive numbers $s_1, \dots, s_n$ so that $s_1 + \cdots s_n = 1000$ and $s_1 \cdots s_n$ is maximized.

Problem 3, Putnam 2009. Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all points P in the plane?

Problem 4, Putnam 2002. Given any five points on a sphere, show that four of them must lie on a closed hemisphere.

Problem 5, Folklore. In any group of six people there are either three mutual acquaintances or three mutual strangers.

Problem 6, *. Find the maximum value of $\sin^4(x) + \cos^4(x)$.

Problem 7, Folklore Give a function $F(n)$ which describes the maximum number of regions that n straight lines divide the plane.

Problem 8, *. If r is a number such that $r + 1/r$ is an integer, show that $r^n + 1/r^n$ is also an integer for all $n = 1, 2, \dots$.

Larson = L.C. Larson, "Problem-Solving Through Problems," Springer, 1983.

* = `https://sites.math.northwestern.edu/~mlerma/problem_solving/putnam/training.html`