**MATH 490, Worksheet #10, Wednesday, April 1, 2020**

**Problem 1.** If $a, b, c$ are positive integers and $a^2 + b^2 = c^2$ show that 3 divides $ab$.

**Problem 2.** How many zeros does 1000! end in?

**Problem 3, Engel.** Three siblings inherit $n$ diamonds with weights $1, \dots, n$ carats. For which values of $n$ may the diamonds be split evenly?

**Problem 4.** Find all primes of the form $n^4 + 4^n$.

**Problem 5, Putnam 1988** If $n \geqslant 3$ is no prime show that there are positive integers $a, b, d$ so that $n = ab + bc + ca + 1$.

**Problem 6.** Show that there are infinitely many primes of the form $4n + 3$.

**Problem 7.** Show that $1 + \frac{1}{2} + \cdots + \frac{1}{n}$ is not an integer for all $n \geqslant 2$.

**Problem 8, Zeitz.** Show any integer greater than 7 is a sum of two relatively prime integers greater than 1.

**Problem 9.** Show that the only non-negative integer solutions of $3^x + 4^y = 5^z$ are $(0, 1, 1)$ and $(2, 2, 2)$.

**Problem 10.** Show that $\lfloor 10^n \pi \rfloor$ ends in 2 for infinitely many $n$.

Engel = A. Engel, "Problem Solving Strategies," Springer, 1997.,

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.