MATH 490, Worksheet #12, Wednesday, April 15, 2020

Problem 1. Find a polynomial with integer coefficients which has $\sqrt{2}+\sqrt{5}$ as a root.

Problem 2. Let a, b, c be the roots of x^3-3x^2+1. Find $a^2+b^2+c^2$ and $a^3+b^3+c^3$.

Problem 3, Putnam 2009. Let f be a non-constant polynomial with positive integer coefficients. For a positive integer n show that $f(n)$ divides $f(f(n)+1)$ exactly when $n=1$.

Problem 4, Putnam 2003. Do there exist polynomials $a(x), b(x), c(y), d(y)$ so that $1+xy+x^2y^2=a(x)c(y)+b(x)d(y)$?

Problem 5, Zeitz. Let $a<b<c$ be numbers so that $a+b+c=6$ and $ab+bc+ca=9$. Show that $0<a<1<b<3<c<4$.

Problem 6. Is there a polynomial $p(x)$ with $xp(x-1)=(x+1)p(x)$ for all x?

Problem 7. Let f be a polynomial with real coefficients such that $f(x)+f'(x)>0$ for all x. Show that $f(x)>0$ for all x.

Problem 8, Putnam 2010. Find all polynomials $p(x), q(x)$ with $p(x)q(x+1)-p(x+1)q(x)=1$.

Problem 9, Conway's Wizards. You overhear two wizards talking on a bus. The first says, "I have a positive number of children whose ages sum to the number of this bus, and whose product is my own age." The second wizard replies, "Perhaps, if you told me the number of your children and your age, I could work out the age of each child." The first wizard says, "No!" "Ah!, then at last I know how old you are," says the second. Now, what is the number of the bus?

Conway = John H. Conway, `https://en.wikipedia.org/wiki/John_Horton_Conway`

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.