MATH 490, Worksheet #13, Wednesday, April 22, 2020

Problem 1. Two players A and B play the following game. A thinks of a polynomial with non-negative integer coefficients. Player B tries to guess the polynomial. B can pick a number and ask A to return the polynomial value there, and then she has another such try. Can B win the game?

Problem 2. A box of 10 matches is on a table. Two players take turns removing 1, 2, or 3 matches. The last player to take a match wins. Which player can always win? What if the game is changed so that the last to take a match loses?

Problem 3, Engel. What is the winning strategy for n matches if the legal moves are removing at least 1 and at most k matches?

Problem 4, Engel. What is the winning strategy for n matches and each player may remove a power of 2 number of matches? ($2^0 = 1$ is a power of 2.)

Problem 5, Engel. In the equation $x^3 + Ax^2 + Bx + C$ two players take turns replacing one of the letters with an integer. Show the first player can always ensure that the resulting cubic polynomial has only integer roots.

Problem 6, Putnam 1965. In a round-robin tournment each of n-players play every other exactly once. Let w_k be the number of wins for player k and l_k the number of that player's losses. If there are no draws show that $\sum w_k^2 = \sum l_k^2$.

Problem 7. Two players alternately put white and black knights on the squares of a chessboard. A knight may not be placed on an occupied square or on a square threatened by an enemy knight. The loser is the one who cannot move anymore. Who wins?

Problem 8, Putnam 2002. A globe is divided up into at least 5 territories so that only 3 territories can share the same boundary point, i.e., there is no "four corners" region like Utah, Colorado, Arizona, and New Mexico share. Two players take turns claiming territories, one claim per turn. A player wins when she has claimed three territories with a common boundary point. Show the first player to claim a territory can always win.

Problem 9, Another from Conway. A 10-digit number ABCDEFGHIJ uses each digit from 0 to 9 exactly once. You are told that A is divisible by 1, AB is divisible by 2, ABC is divisible by 3, and so on until $ABCDEFGHIJ$ is divisible by 10. What is the number?

Engel = A. Engel, "Problem Solving Strategies," Springer, 1997.,