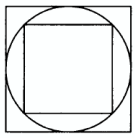
MATH 490, WORKSHEET #2
WEDNESDAY, JANUARY 22, 2020

Problem 1, Zeitz. Inscribe a circle in a square, then inscribe a square in the circle. What is the ratio of areas of the squares?



Problem 2, Zeitz. How many subsets of $\{1, \ldots, 30\}$ have the property that the sum of the elements is greater than 232?

Problem 3, Putnam 1998. Let $P = (4, 3)$. Find the minimal perimeter of a triangle PQR with Q on the line $y = x$ and R on the x-axis.

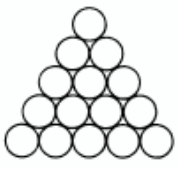
Problem 4, Zeitz. There are 2020 points arranged evenly on a circle. Each point is assigned a number so that that number is the average of the numbers assigned to its two nearest neighbors. Show that each point assigned the same number.

Problem 5, Putnam 2019. Find all possible values of $A^3 + B^3 + C^3 - 3ABC$ for non-negative integers A, B, C.

Problem 6, Putnam 1980. Find

$$\int_0^{\pi/2} \frac{1}{1 + \tan^3(x)} dx$$

Problem 7, Larson. 15 coins are arranged as follows, colored either black or white. Show there are coins of the same color whose centers are the vertices of an equilateral triangle.



Larson = L.C. Larson, "Problem-Solving Through Problems," Springer, 1983.

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.