MATH 490, WORKSHEET #3
WEDNESDAY, JANUARY 29, 2020

Problem 1, Larson. A room is 30 feet long, 12 feet high, and 12 feet wide on either end. On one end an ant is one foot below the ceiling in the middle and there is a piece of food one foot up from the floor in the middle of the opposite wall. If the ant has only enough energy to crawl 40 feet, can it make it to the food?

Problem 2, Zeitz. How many ordered pairs (x, y) with $0 < x, y < 1$ are there so that both $3x + 7y$ and $5x + y$ are integers?

Problem 3, Larson. Let $0 < a < b$ what is the probability of picking two numbers in $[0, b]$ independently at random which are at least distance a apart?

Problem 4, Putnam 2005. Find a polynomial $P(x, y)$ so that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a.

Problem 5, Zeitz. 2020 points are arranged uniformly on a circle. Each point is assigned a number which happens to be the average of the numbers assigned to its two nearest neighbors. Show that each point is assigned the same number.

Problem 6, Putnam 2008. A circle is centered at a point (x, y) so that at least one of x and y is irrational. How many points with rational coordinates can lie on the circle?

Problem 7, Zeitz. Show that the plane may be tiled using infinitely many squares and infinitely equilateral triangles of the same side length. Show this cannot be done using infinitely many squares and at least one, but only finitely many, equilateral triangles.

Problem 8, Putnam 2006. Find the volume of the set of points (x, y, z) satisfying

$$(x^2 + y^2 + z^2 + 8)^2 \leqslant 36(x^2 + y^2).$$

Larson = L.C. Larson, "Problem-Solving Through Problems," Springer, 1983.

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.