MATH 490, WORKSHEET #4
WEDNESDAY, FEBRUARY 5, 2020

Problem 1. Given an equilateral triangle of side length 1, show that among any five points inside the triangle there are two that are distance less than $\frac{1}{2}$ apart.

Problem 2. Among five lattice points there are two whose midpoint is also a lattice point. Show this is false for four points.

Problem 3, Larson. For x a real number show that at least one of $x, 2x, \ldots, (n-1)x$ differs from an integer by at most $\frac{1}{n}$.

Problem 4, Zeitz. Color the plane using 3 colors. Show there are two points exactly distance 1 apart with the same color.

Problem 5, Putnam. Among any 5 integers there are 3 whose sum is divisible by 3.

Problem 6, Zeitz. A chess player practices for a tournament by playing at least one game per day for 8 weeks, but no more than 11 games per week. Show there is some set of consecutive days where she plays exactly 23 games.

Problem 7, Zeitz. For a positive integer n, show there is some positive multiple whose decimal expansion only contains the digits 0 and 7.

Problem 8, Erdős. Let S be a subset of $n+1$ elements of $\{1, \ldots, 2n\}$. Show there are a, b in S, distinct, so that a divides b.

Erdős = Paul Erdős. `https://en.wikipedia.org/wiki/Paul_Erdos`

Larson = L.C. Larson, "Problem-Solving Through Problems," Springer, 1983.

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.