MATH 490, Worksheet #6, Wednesday, February 19, 2020

Problem 1, Zeitz. Suppose that finitely many coins are arranged on a table, each with a different diameter. Show there is a coin which is tangent to at most five others.

Problem 2. Finitely many points in the plane are colored red and blue. Suppose that any line segment that joins points of the same color contains a point of the opposite color. Show that all points lie on the same line.

Problem 3, Engel. An odd number of people each has a dodgeball. They stand in a way such that all mutual distances are different. Each person throws their ball at their nearest neighbor. Show at least one person survives, nobody is hit by more than five dodgeballs, and the dodgeballs do not cross paths.

Problem 4, Zeitz. An infinite chessboard contains a positive integer in each square. Suppose each integer is the average of its north, south, east, and west neighbors. Show that all there integers are equal.

Problem 5, Engel. All plane sections of a solid are circles. Show that solid is a ball.

Problem 6, Engel. The equation $x^2 + y^2 = 3(z^2 + w^2)$ has no solutions where x, y, z, w are all positive integers.

Problem 7, Engel. A collection of points in the plane has the property that any triangle formed by any three points has area at most 1. Show that all points are contained in a triangle of area at most 4.

Problem 8, Sylvester. Suppose a collection of finitely many points in the plane has the property that three points lie on every line connecting any two. Show the points must all lie in the same line.

Engel = A. Engel, "Problem Solving Strategies," Springer, 1997.

Sylvester = Sylvester–Gallai Theorem, `https://en.wikipedia.org/wiki/Sylvester\T1\textendashGallai_theorem`

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.