MATH 490, Worksheet #7, Wednesday, February 26, 2020

Problem 1, Zeitz. Among the integers from 1 to $1,000,000$ are there more numbers which can be written as the sum of a perfect square and a (positive) perfect cube or more numbers which cannot be written as such?

Problem 2, Engel. Each face of a cube is painted a different color from six possible colors. How many distinct colorings of the cube are there?

Problem 3, Engel. How many positive integers $\{1, 2, \ldots, 1000\}$ are divisible by 7 or 11?

Problem 4, Zeitz. On an 8×8 chessboard how many ways are there to select two squares so that they are not in the same row or column?

Problem 5, Zeitz. 10 dogs come across 8 identical biscuits. Show that $C(17, 8)$ is the total number of ways the biscuits may be consumed. (Dogs don't share biscuits!)

Problem 6, Putnam 1992. Given n distinct real numbers, consider the set A of all averages of pairs of numbers in the set. What is the smallest cardinality of A?

Problem 7, Putnam 2003. How many ways are there to write a positive integer n as a sum of positive integers $a_1 \leqslant a_2 \leqslant \cdots \leqslant a_k \leqslant a_1 + 1$?

Problem 8, ICMC 1997. In Indiana, license plates consist of 2 digits followed by a letter and then 4 more digits. Find the probability of getting a license plate in which the last four digits are nondecreasing.

Problem 9, Engel. How many subsets of $\{1, \ldots, n\}$ contain no two consecutive numbers?

Engel = A. Engel, "Problem Solving Strategies," Springer, 1997.

ICMC = Indiana Collegiate Mathematics Contest. `http://sections.maa.org/indiana/ICMC.php`

Zeitz = P. Zeitz, "The Art and Craft of Problem Solving" 2 ed. Wiley, 2007.