

The VC-dimension of a subset of an Abelian group is a measure of structure relative to the ambient group; a structured subset has low VC-dimension while an unstructured subset has high VC-dimension.

Let $\Gamma \subseteq \mathbb{F}_q^\times$ be a multiplicative subgroup of a finite field. We will investigate its VC-dimension when considered as a subset of the additive group $(\mathbb{F}_q, +)$. A particular case of interest is the set of quadratic residues $\Gamma = \{x^2 : x \in \mathbb{F}_q^\times\}$.

We will present conjectures concerning this VC-dimension which are based on the heuristic expectation that additive and multiplicative structure must be loosely correlated. We will provide some partial progress towards them, based on the Weil bound for multiplicative character sums. This is joint work with Brian McDonald and Emmett Wyman.

The quadratic residues are an example of a pseudorandom subset of $(\mathbb{F}_q, +)$, and this raises interesting questions about what happens for truly random subsets in more general groups. If time permits, we will present some of these questions and discuss ongoing work with Brad Rodgers which addresses some of them.