Let p be a prime, and let $\mathcal{A}$ be a set of residues of the sequence $1!, 2!, 3!, \ldots$ modulo p. We prove

$$|\mathcal{A}| \geqslant (\sqrt{2} + o(1))\sqrt{p}.$$

Now consider an interval $\mathcal{I} \subseteq \{0, 1, \ldots, p-1\}$ of length $N > p^{7/8+\varepsilon}$ and denote by $\mathcal{A}_{\mathcal{I}}$ the set of residues modulo p it produces. Then we prove

$$|\mathcal{A}_{\mathcal{I}}| > (1 + o(1))\sqrt{p}.$$

Tools used are the results from Algebraic Geometry as black boxes and simple combinatorial observations.

This is a joint work with Alexandr Grebennikov, Arsenii Sagdeev, Aliaksei Vasilevskii, and available on arXiv: https://arxiv.org/abs/2204.01153.