How many rational points with denominator of a given size lie within a specified distance from a compact, "non-degenerate" manifold? A precise answer to this question has significant implications for a host of problems including Diophantine approximation on manifolds. In this talk, I will describe how the analytic and geometric properties of the manifold critically influence this count, and provide a heuristic for it. Further, I will talk about recent work which leverages Fourier analytic methods to establish the desired asymptotic for manifolds satisfying a strong curvature condition. At the heart of the proof is a bootstrapping argument that combines Poisson summation, convex duality, and oscillatory integral techniques.