

I will present joint work with Maxim Gerspach on lower-order terms in Selberg's central limit theorem. Specifically, we establish precise asymptotic formulas for the third moment of both the real and imaginary parts of the logarithm of the Riemann zeta function. Our results are conditional on the Riemann Hypothesis, Hejhal's triple correlation, and a new conjecture that describes the interaction between prime powers and Montgomery's pair correlation function. To support this conjecture, which we refer to as the "twisted" pair correlation conjecture, we prove it unconditionally in a limited range and under the Hardy-Littlewood conjecture in a larger range.