

Quadratic Gauss sums are defined as $\sum_{x \bmod q} \exp(2\pi i x^2)$. One can view these as finite field analogue of Gaussian integrals, besides they are quite useful, for example one can derive quadratic reciprocity from properties of Quadratic Gauss sums. As a natural Generalization Kummer was interested in the 19th century in the properties of Cubic Gauss sums, defined as $\sum_{x \bmod q} \exp(2\pi i x^3)$. In his attempts at establishing a formula for these higher order Gauss sums Kummer noticed a numerical bias in the signs of these (real-valued) sums. The existence of this bias perplexed number theorists for a long time. Patterson was the first to realize that cubic Gauss sums can be realized as coefficients of very exotic (weight 1/3) automorphic forms. This led to a conjectural explanation of the bias that Kummer observed, the existence of which was finally confirmed (conditionally on the Generalized Riemann Hypothesis) in recent-ish work of the speaker with Alex Dunn. I will discuss the circle of ideas surrounding these cubic Gauss sums, and the reasons for why they remain both interesting and mysterious.