Let $\alpha$ be a positive non-integer real number. We consider the values $n^\alpha$ modulo 1 with $n < N$ and place them in order on the interval $[0, 1]$. It is tempting to conjecture that these values should look like a set of random points, therefore the gap distribution (at the scale $1/N$) should be Poisson.

It turns out however, as proven by Elkies and McMullen, that the gap distribution exists and is not Poisson when $\alpha = 1/2$. This is the only exponent for which the existence of the gap distribution is proven. The proof of Elkies-McMullen and the later proof of Browning-Vinogradov are dynamical in nature. We will present a self-contained and elementary proof based on the circle method.

I will discuss the differences between the two proofs, time permitting. Joint work with Niclas Technau.