**MA 266** (Fall 2020) **Midterm 1**
Instructor: Otávio Menezes Tuesday, September 29, 2020

**Instructions**:

- You may use a calculator, but you must show all your work in order to receive credit.
- Be sure to erase or cross out any work that you do not want graded. Circle your final answer.
- When necessary, round answers to two decimal places.

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Points: | 15 | 20 | 15 | 20 | 15 | 15 | 100 |
| Score: | | | | | | | |

1. Consider the differential equation $y' = y(1-y)$.

   (a) Show that, for any choice of constant $C$, the function (5)

   $$y(t) = \frac{e^t}{e^t + C}$$

   solves the differential equation.

   (b) Is there any solution that is not of the form $y(t) = \frac{e^t}{e^t+C}$? (5)

   (c) Solve the initial value problem (5)

   $$y' = y(1-y), \quad y(0) = 3.$$

2. Consider the following family of initial value problems, indexed by $h > 0$: (20)

$$\frac{dy}{dt} = y(3 + y) + h, \quad y(0) = 1.$$

For which values of $h$ does $\lim_{t\to\infty} y(t) = +\infty$?

3. Consider the differential equation

$$(4x^2y^3 + 9x^3y^2)y' + 3xy^4 + 12xy^3 = 0.$$

(a) Show that this equation is not exact. (5)

(b) Show that it becomes exact after being multiplied by $x$. (5)

(c) Find the general solution. (5)

4. A 25-liter tank is filled with water and 3 kg of salt. A salty solution with concentration 5 kg of salt per liter is added to the tank at a rate of 2 liters per minute. Through a separate spout, water is allowed to exit the tank at a rate of 2 liters per minute so that it does not overflow. We may assume the water in the tank is always well-mixed. How much salt is in the tank after 5 minutes? (Round your answer to two decimal places.) (20)

5. Consider the differential equation

$$x^2y'' + 2xy' - 12y = 0$$

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(a) Show that the substitution $v = \ln x$ transforms the equation into a linear equation with constant coefficients. (8)

(b) Find the general solution. (7)

6. Solve the initial value problem (15)

$$9y'' + 6y' + 4y = 0, \quad y(0) = 3, y'(0) = 4.$$