

Instructions:

- Books and calculators are allowed.
- Show all your work to receive credit.
- In your Gradescope submission, start a new page for each problem.

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|-----------|----|----|----|----|----|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | Total |
| Points: | 20 | 20 | 20 | 20 | 20 | 100 |
| Score: | | | | | | |

1. A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ and initial velocity $v_0 = -10\text{m/s}$. Find the amplitude and period of the resulting motion. (20)

2. Solve the initial value problem

(20)

$$y'' + 4y = 3x; \quad y(0) = 1, y'(0) = 2.$$

3. Find the general solution of the system

(20)

$$x'' = -5x + 2y, y'' = 2x - 8y.$$

4. Find two linearly independent solutions of the system (20)

$$\mathbf{x}'(t) = \begin{bmatrix} 5 & -1 & 1 \\ 1 & 3 & 0 \\ -3 & 2 & 1 \end{bmatrix} \mathbf{x}(t).$$

Given: the characteristic polynomial of the matrix is $(\lambda - 3)^3$.

5. Solve the initial value problem

(20)

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}(t), \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$