1.0 ABOUT THE COURSE

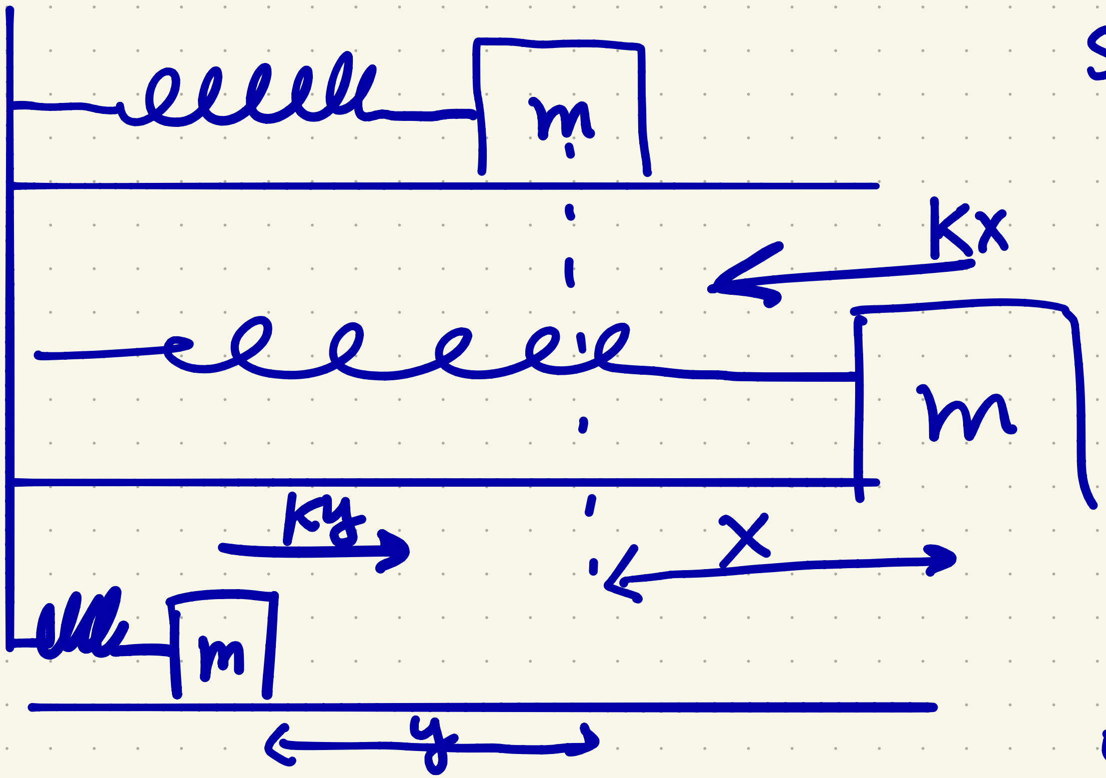
PRE-REQUISITES:

→ calculus : functions, derivatives as rates of change

→ linear algebra (optional): vector spaces, linear maps, matrix mult, linear independence

SOME PROBLEMS WE'LL SOLVE IN THE COURSE

1) HARMONIC OSCILLATOR (sections 3.4, 3.5, 3.6)

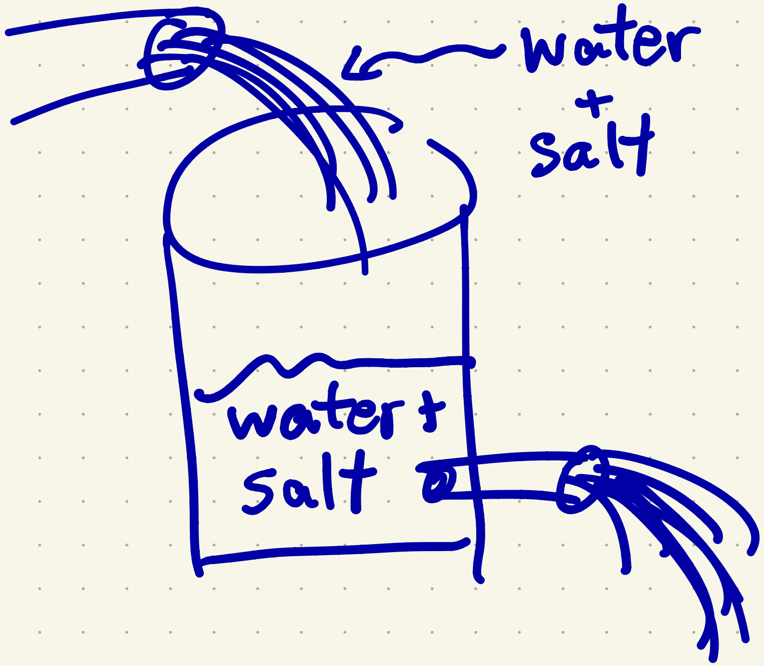


mass m attached to a spring

Force of magnit. Kx towards equilibrium position

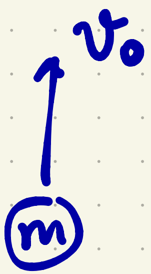
Q: given the initial position and velocity, predict the movement, even when there's frict.

2) Concentration (section 1.5)



GIVEN THE INITIAL AMOUNT OF SALT AN THE IN/OUT RATES, PREDICT THE FUTURE AMOUNTS OF SALT.

3) Kinematics (sec 1.2, 2.3)

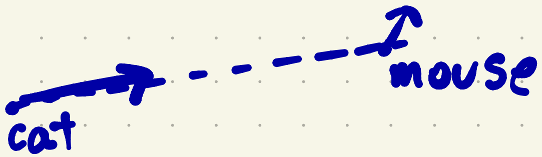


A ball is thrown upward at speed v_0.

How high does it go?

What if there is air resistance?

4) Pursuit curves



Given the path followed by the mouse and the speeds of the cat at the mouse, find the path followed by the cat.

1.1 DIFFERENTIAL EQUATIONS AND MATHEMATICAL MODELS

↪ DIFF EQ
↪ IVP
↪ SOLUTION
↪ GENERAL SOL.

EQUATION: a <u>possible</u> relationschip between some unspecified mathematical objects

Examples: $x+3=5$ algebraic eqs $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ 6\end{bmatrix}$ matrix eq

$y^2+3y+2=93$

$\cos t + e^t = \sin t$

DIFFERENTIAL EQUATION: a possible rel between some unspecified functions and their derivatives

<u>EXAMPLE</u>: $y'(t)=0, \ t>0$ ← a function whose derivative is 0 at all points

$y'=y$ ← a function that equals its on derivative

$y'=y(1-y)$ $xy'=y$

A SOLUTION to a diff. eq. is a function for which the relation is true.

$y'(x) = 0$ any constant function is a solution

$y' = y$

$y(x) = e^x$ is a solution

$y(x) = 0$ is a solution

$y(x) = Ce^x$ is a solution for any number C

$y'' = -y$

$y(x) = \sin x$ is a solution

$y(x) = \cos x$ is also a solution

$y(x) = \sin x + \cos x$ is also a solution

$y' = y(1-y)$

$y(x) = 0$ is a solution

$y(x) = 1$ is a solution

$$\begin{cases} s' = c \\ c' = -s \\ s(0) = 0 \\ c(0) = 1 \end{cases} \qquad \begin{cases} s' = c \\ c' = +s \\ s(0) = 0 \\ c(0) = 1 \end{cases}$$

A GENERAL SOLUTION to a diff. eq. is a formula with some free parameters that generates ALL solutions when numeric values are attributed to the parameters.

EX 1) $y'=0$ General solution $y=C$ ("one-parameter family")

$y=2C$ is the same gen. sol.

2) $y'=y$ General sol. $y(t)=Ce^t$

3) $y''=y$ General sol. $y(t)=A\cos t+B\sin t$ "two-parameter fam"

4) $y'=y(1-y)$ Sol. $y(t)=\dfrac{e^t}{e^t+C}$

Check that this formula gives an actual solution.

$$y' = y(1-y)$$

$$\left(\frac{e^t}{e^t+C}\right)' \stackrel{?}{=} \frac{e^t}{e^t+C}\left(1-\frac{e^t}{e^t+C}\right)$$

$$\left(\frac{e^t}{e^t+c}\right)' \overset{?}{=} \frac{e^t}{e^t+c}\left(1-\frac{e^t}{e^t+c}\right)$$

$$\frac{(\cancel{e^t}+c)e^t - e^t\cancel{e^t}}{(e^t+c)^2} = \frac{e^t}{e^t+c}\,\frac{c}{e^t+c} \quad ✓$$

why is this not the general solution?

It does not generate $y=0$.