Numerical Analysis

Instructor: Professor Steven Dong
Course Number: MA 51400
Credits: Three
Time: 3:30–4:20 PM MWF

Catalog Description

(CS 51400) Iterative methods for solving nonlinear; linear difference equations, applications to solution of polynomial equations; differentiation and integration formulas; numerical solution of ordinary differential equations; roundoff error bounds.

Introduction To Probability

Instructor: Professor Gregery Buzzard
Course Number: MA 51900
Credits: Three
Time: 10:30–11:20 AM MWF

Catalog Description

(STAT 51900) Algebra of sets, sample spaces, combinatorial problems, independence, random variables, distribution functions, moment generating functions, special continuous and discrete distributions, distribution of a function of a random variable, limit theorems.

Introduction to Partial Differential Equations

Instructor: Professor Arshak Petrosyan
Course Number: MA 52300
Credits: Three
Time: 9:00–10:15 AM TTh

Catalog Description

First order quasi-linear equations and their application to physical and social sciences; the Cauchy–Kovalevsky theorem; characteristics, classification, and canonical form of linear equations; equations of mathematical physics; study of the Laplace, wave and heat equations; methods of solution.

Functions Of A Complex Variable I

Instructor: Professor Plamen Stefanov
Course Number: MA 53000
Credits: Three
Time: 1:30–2:20 PM MWF

Catalog Description

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics. More mathematically rigorous than MA 52500.

Probability Theory II

Instructor: Professor Jing Wang
Course Number: MA 53900
Credits: Three
Time: 2:30–3:20 PM MWF

Catalog Description

(STAT 53900) Convergence of probability laws; characteristic functions; convergence to the normal law; infinitely divisible and stable laws; Brownian motion and the invariance principle.

Real Analysis And Measure Theory
Instructor: Professor Monica Torres
Course Number: MA 54400
Credits: Three
Time: 11:30–12:20 PM MWF

Description

Abstract measure spaces, construction of Lebesgue measure, properties of measurable functions, Lebesgue integral, Egorov's theorem, Lusin's theorem, Lebesgue dominated convergence theorem, L^p spaces, Fubini's theorem, convolutions in $\mathbb{R}^n$, Vitali's covering theorem, Lebesgue differentiation theorem, functions of bounded variation, absolutely continuous functions, diffentiation of monotone functions, Fundamental Theorem of Calculus.

References: I will provide notes for the class based on the following references: Modern Real Analysis (by W.P. Ziemer and with contributions by M. Torres) and Measure and Integral (by R. L. Wheeden and A. Zygmund). Other references include: An Introduction to Measure Theory (by Terence Tao), Real Analysis (by E.M. Stein and R. Shakarchi).

Introduction To Abstract Algebra
Instructor: Professor Saugata Basu
Course Number: MA 55300
Credits: Three
Time: 3:30–4:20 PM MWF

Catalog Description

Group theory: Sylow theorems, Jordan-Holder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

Linear Algebra
Instructor: Professor Jeremy Miller
Course Number: MA 55400
Credits: Three
Time: 2:30–3:20 PM MWF

Description

Review of basics: vector spaces, dimension, linear maps, matrices determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

Abstract Algebra I
Instructor: Professor Jaroslaw Wlodarczyk
Course Number: MA 55700
Credits: Three
Time: 4:30–5:45 PM TTh

Description

The course covers material from the text *Introduction to Commutative Algebra* by M. F. Atiyah and I. G. Macdonald. In particular, I plan to cover the following topics:

- Rings and ideals
- Zariski topology
- Modules
- Rings and modules of fractions
- Primary decomposition
- Integral dependence and valuations

- Chain conditions
- Noetherian rings
- Artin rings
- Discrete valuation rings and Dedekind domains
- Completions and dimension theory

Brief treatments of other topics illustrating connections between algebraic notions and affine algebraic geometry will be included as time permits. An important part of the class will be solving problems from the Atiyah-Macdonald book.

Textbook:

Introduction to Commutative Algebra by M. F. Atiyah and I. G. Macdonald. available via the Purdue library at: https://doi.org/10.1201/9780429493638

Introduction To Differential Geometry And Topology
Instructor: Professor Ben McReynolds
Course Number: MA 56200
Credits: Three
Time: 12:30–1:20 PM MWF

Catalog Description

Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in E3., Gaussian curvature; two dimensional Riemannian geometry; Gauss-Bonnet and Poincare theorems on vector fields.

Elementary Topology
Instructor: Professor Manuel Rivera
Course Number: MA 57100
Credits: Three
Time: 9:30–10:20 AM MWF

Catalog Description

Fundamentals of point set topology with a brief introduction to the fundamental group and related topics, topological and metric spaces, compactness, connectedness, separation properties, local compactness, introduction to function spaces, basic notions involving deformations of continuous paths.

Numerical Solution of Ordinary Differential Equations
Instructor: Professor Di Qi
Course Number: MA 57300
Credits: Three
Time: 8:30–9:20 AM MWF

Description

This course meant to introduce graduate students with various background to the fundamentals and applications of numerical methods essential for solving differential equations and dynamical systems. The course will cover key concepts with a balance between theoretical analysis and practical implementation. Solutions of ordinary differential equations will be discussed first, including single and multistep methods for initial value problems, and Runge-Kutta schemes, iterative methods for solving large systems of equations, convergence and stability, and methods for stiff problems such as exponential temporal integrators and multigrid iterative solvers. Numerical solutions to stochastic differential equations and sequential Monte Carlo sampling strategies will then be explored. Numerical methods for dynamical systems will be discussed including Hamiltonian systems, invariant sets and chaotic attractors. The course will continue to applications in finance using the time stepping schemes to solve boundary and eigenvalue problems of partial differential equations, and building data-driven strategies for solving time-dependent sequential predictions.

Audience:: The course should be suitable to any graduate student in applied and computational mathematics, physics, engineering, as well as related fields involving numerical computing and computer programming.

Algebraic Number Theory
Instructor: Professor Daniel Le
Course Number: MA 58400
Credits: Three
Time: 1:30–2:20 PM   MWF

Description

Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.

Algebraic Geometry I
Instructor: Professor Takumi Murayama
Course Number: MA 59500AGI
Credits: Three
Time: 1:30–2:20 PM   MWF

Description

This course is the first course in a two semester introductory sequence in algebraic geometry. Algebraic geometry is the geometric study of solutions to systems of polynomial equations. Algebraic geometry has interactions with many other fields of mathematics, including commutative algebra, algebraic topology, number theory, several complex variables, and complex geometry.

This first course will mainly focus on the theory of algebraic varieties over algebraically closed fields. The following topics will be covered.

- Affine varieties.
- Projective varieties.
- Morphisms of algebraic varieties.
- Rational maps between algebraic varieties.
- Lüroth's theorem.
- Nonsingular varieties.
- Intersections in projective space, Bézout's theorem.
- Elimination theory, resultants.
- Semicontinuity of the dimension of fibers.
- Grassmannians.
- Every smooth cubic surface has exactly 27 lines.
- Sheaves of Abelian groups.
- The definition of a scheme.

Prerequisites:

- MA 55300 and MA 55400.
- MA 57100.
- Commutative Algebra at the level of *Introduction to commutative algebra* by Atiyah and Macdonald (often taught under MA 55700). Can be taken concurrently with this course.

MA 56200 and 57200 are recommended.

Text: Typed course notes by the instructor will be provided. Notes from the Fall 2024 iteration of the course are available here:

https://drive.google.com/file/d/1ENjHO5_MVdXFOy_6iJP5MEFofSLa7Zxm

Optional texts: All texts listed below have free access options for Purdue students.

For algebraic varieties:

1. *Algebraic geometry: A first course* by Joe Harris. Available here: https://doi.org/10.1007/978-1-4757-2189-8.
2. Chapter I of *Algebraic geometry* by Robin Hartshorne. Available here: https://doi.org/10.1007/978-1-4757-3849-0.
3. *Undergraduate algebraic geometry* by Miles Reid, available here https://homepages.warwick.ac.uk/staff/Miles.Reid/MA4A5/UAG.pdf.
4. *Basic algebraic geometry 1* (third edition) by Igor R. Shafarevich. Available here: https://doi.org/10.1007/978-3-642-37956-7.

For sheaves and schemes:

5. *Éléments de géométrie algébrique* by Alexander Grothendieck and Jean Dieudonné. Available here: http://www.numdam.org/item/PMIHES_1960__4__5_0 (and then keep clicking the "followed-by" links).
6. *Eléments de géométrie algébrique I* (second edition) by Alexander Grothendieck and Jean Dieudonné. Available for short term loan here: https://n2t.net/ark:/13960/t42s6kw4b.

Applied Etale Cohomology
Instructor: Professor Deepam Patel
Course Number: MA 59500ECMEC
Credits: Three
Time: 12:00–1:15 PM TTh

Description

In this course, we will discuss applications of Etale Cohomology to questions in number theory. Specifically, we will discuss applications to bounding exponential sums, and equi-distribution theorems. Since the main focus of the course will be towards applications, we will only state (and not prove) some of the foundational theorems in etale cohomology. Instead, we will focus ondeveloping the tools needed for the aforementioned applications e.x. computations in local/global monodromy, and (if time permits) microlocal analysis in charateristic p.

Prerequisites: I will assume basic Algebraic Geometry at the level of Hartshorne's book. I will also assume some basic Algebraic Topology including the basics of singular homology and cohomology.

Grobner Bases Commutative Algebra
Instructor: Professor Giulio Caviglia
Course Number: MA 59500GB
Credits: Three
Time: 8:30–9:20 AM MWF

Description

This is an introductory course on Grobner Bases. We will focus on their applications to Commutative Algebra and cover both theoretical and computational aspects. The central topics will be: graded free resolutions and their homological invariants, generic initial ideals, determinantal ideals and toric ideals. I will loosely follow the book: Grobner Bases in Commutative Algebra by V. Ene and J. Herzog.

Prerequisites: This course is intended to graduate students with a certain familiarity with the basic concepts of Commutative Algebra. A previous knowledge of Commutative Algebra, for instance the material covered in MA55700 and MA55800, will be useful although not required.

Homological Algebra
Instructor: Professor Ralph Kaufmann
Course Number: MA 59500HAM
Credits: Three
Time: 10:30–11:20 AM   MWF

Description

Homological algebra is an indispensable tool in many areas of mathematics, such as topology, algebra, algebraic geometry and mathematical physics related to string theory.

We will start with the basic theory of derived functors like Tor and Ext, and then move to localization, roof calculus derived categories and triangulated categories. We will end the regular topics by discussing model categories. If time permits, and depending on the audience, we will treat additional topics like exceptional collections, stability conditions and perverse sheaves. Throughout we will use examples from topology and algebraic geometry. The course will go by its own lecture notes, but the main sources are:

S.I. Gelfand and Yu. I. Manin *Methods of Homological Algebra* Springer 1996 Charles A. Weibel *An introduction to homological algebra.* Cambridge Studies in advanced mathematics. Cambridge University Press 1997.

Large Deviations
Instructor: Professor Jonathon Peterson
Course Number: MA 59500LD
Credits: Three
Time: 9:30–10:20 AM MWF

Description

Large deviation theory is, informally, the study of the asymptotics of probabilities of unlikely events. As an example, while the law of large numbers says that the empirical mean of the sum of a large number of i.i.d. random variables is typically close to the expected value of the random variables, large deviation theory gives a framework for evaluating the asymptotics of the probability the empirical mean is close to a number different than the expected value. Large deviation theory not only gives understanding of how small probabilities of rare events are, but often also gives an insight into exactly how rare events arise (that is, given that a rare event occurred, what is the most likely way that this rare event happened). Large deviation theory is helpful for anyone working in probability theory but also has applications in statistics and other applied areas where one needs some understanding or control of rare events.

Prerequisites: 538 (532 and 539 will also be helpful but are not required).

Introduction to Microlocal Analysis
Instructor: Professor Plamen Stefanov
Course Number: MA 59500PDO
Credits: Three
Time: 11:30–12:20 PM MWF

Description

This course is an introduction to some of the most important topics in Microlocal Analysis, which, roughly speaking, studies functions or distributions and linear operators in the "phase space," i.e., at points and (co)-directions. I will start with a review of distributions, the Fourier Transform and introduction of Wave Front Sets. Then I will introduce the pseudo-differential operators in an open domain and develop the calculus of such operators: sums, compositions, adjoints, boundedness in Sobolev spaces, etc. Then I will show how to build a parametrix of an elliptic operator, and what it is good for: local solvability of elliptic PDEs, elliptic regularity and Fredholm properties of elliptic operators. Application to tomography (integral geometry) will be presented as well. The second part of the course will deal with hyperbolic equations, like the wave equation with variable coefficients. I will explain the geometric optics construction and that will serve as an example of a Fourier Integral Operator (FIO) even though the theory of FIOs is beyond the scope of this course. One of the fundamental results of the theory is Hormander's theorem of propagation of singularities, which will be presented in the context of the wave equation first.

Prerequisites for the course are: familiarity with distributions and the Fourier Transform (that we will review anyway), linear operators in Banach and Hilbert spaces, basic PDE theory.

Methods of Linear and Nonlinear Partial Differential Equations I

Instructor: Professor Matthew Novack

Course Number: MA 64200

Credits: Three

Time: 10:30–11:45 AM TTh

Description

This is the first semester in a one-year course on the theory of PDEs. The course will begin with the basic tools needed to analyze linear and nonlinear PDEs (distributions, Sobolev spaces, and the Fourier transform). Afterwards we will present the basic variational tools for analyzing second order elliptic equations (minimization of energy, Lax-Milgram theorem). Finally, regularity theory for second order elliptic equations will be covered (maximum principles, Harnack inequality, Schauder estimates, and Sobolev estimates).

We will draw from the texts *Methods of Applied Mathematics* by Arbogast and Bona and *Elliptic Partial Differential Equations of Second Order* by Gilbarg and Trudinger, although I will produce class notes which will be available on my webpage. A good knowledge of measure theory and a rudimentary knowledge of normed vector spaces will be necessary; notes for this material will also be available on my webpage. All students considering research in PDEs are encouraged to register.

Boundary Behavior of Holomorphic Functions
Instructor: Professor Steven Bell
Course Number: MA 69300BH
Credits: Three
Time: 2:30–3:20 PM MWF

Description

The only prerequisites for this course are MA 530 and a rudimentary understanding of $L^2[a, b]$ as a Hilbert space.

After MA 530, there are many directions a second course in complex analysis could take. One could dwell on the fruitful interaction of real analysis and complex analysis in the study of the boundary behavior of holomorphic functions and the Cauchy integral. One could study one-dimensional complex manifolds (Riemann surfaces) or functions of several complex variables. Another avenue could be to study more subtle questions in conformal mapping about the boundary behaviour of conformal maps or the new questions that arise about conformal mapping of multiply connected domains.

I will endeavor to present substantial results in all of these areas guided by a study of the Cauchy transform made possible by the remarkable discovery in 1978 by N. Kerzman and E. M. Stein that the Cauchy Transform is nearly a *self adjoint* operator when viewed as an operator on L^2 of the boundary. This fundamental result caused a shift in the bedrock of complex analysis. It has allowed the classical objects of potential theory and conformal mapping in the plane such as the Poisson kernel, the Bergman kernel, and the Szegő kernel to be constructed and analyzed in new and very concrete terms.

I will take a distinctly PDE approach to complex analysis in one variable that originated in Lars Hörmander's classic work on several complex variables from the 1960's. There, solving the inhomogeneous Cauchy-Riemann equations was central. In the plane, this can be done rather directly using the Cauchy integral and ideas from Kerzman and Stein. Exploring this line of thought will allow me to present similar material in several complex variables from time to time.

Reference material

• *The Cauchy transform, potential theory, and conformal mapping*, 2nd edition, Steven R. Bell, CRC Press, 2015.

• *An introduction to complex analysis in several variables*, Lars Hörmander, North Holland, 1973.

• *The Cauchy kernel, the Szegő kernel, and the Riemann mapping function*, N. Kerzman and E. M. Stein, Math. Ann. **236** (1978), 85–93.