Math 460: Homework # 2. Problems 1-4 due Sept. 4; rest due Sept.11.

1. Use Geometer's Sketchpad to construct a triangle, along with the following

   (a) its circumcenter (labeled O)

   (b) its incenter (labeled I)

   (c) its orthocenter (labeled H)

   (d) its centroid (labeled G)

   (e) the line through O and H.

   Hide all the lines used in constructions (a)-(d). Print out a copy, then change the shape of the triangle and print another copy. The line through O and H has a special property that should be obvious from your pictures–what is it? (You do not need to prove anything for this problem.)

2. (Use Geometer's Sketchpad) Start with a triangle ABC. Let D and E be points on the segments AC and BC, respectively, with DE parallel to AB. Let F be the intersection of the segments DB and AE, and let G be the intersection of AB with the ray CF. What special property does G have? Display a measurement which shows it has this property. Print the picture, then change the shape of the triangle, check that G still has this property, and print the new picture. You do not have to prove anything for this problem.

3. (See Figure 1) Given: UV is parallel to AB, UW is parallel to BC, and VW is parallel to AC. To prove: $\triangle AWU \cong \triangle WBV$.

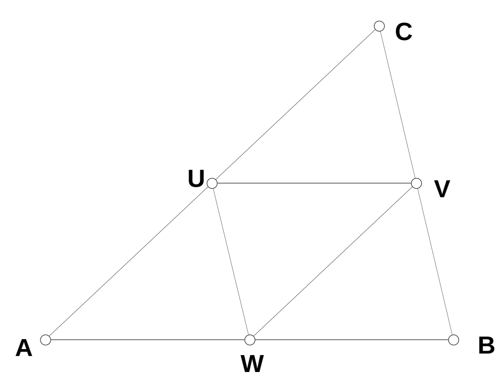

Figure 1

4. (See Figure 2) Given: $\angle 1 = \angle 2$. To prove: $\dfrac{AD}{AC} = \dfrac{BD}{BC}$

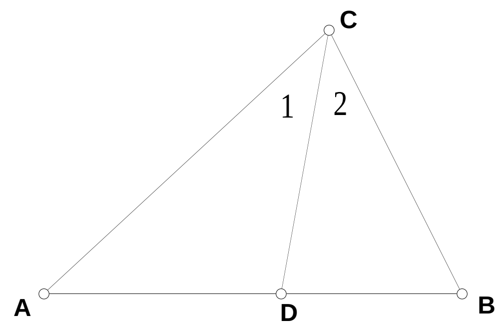

Figure 2

5. Given a quadrilateral $ABCD$ with $AB = BC$ and $CD = AD$, prove that the diagonals AC and BD are perpendicular.

6. (See Figure 3) Given $MK = MQ, \angle K = \angle Q$, PM is perpendicular to MK, and LM is perpendicular to MQ, prove $RS = TS$. (Hint: Use what was shown in problem 5 of the first assignment.) **Warning:** Although it is true that equals subtracted from equals give equals, the same idea is *not* valid for congruence (it is not always true that congruent triangles subtracted from congruent triangles give congruent triangles.)

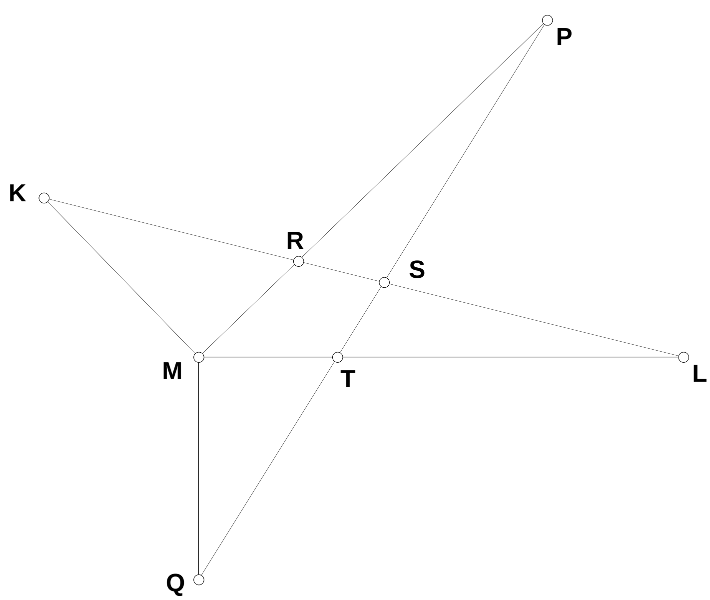

Figure 3

7. Let $ABCD$ be a quadrilateral, and let M, N, P, and Q be the midpoints of the sides. Prove that $MNPQ$ is a parallelogram.

8. (See Figure 4) Given: DE is parallel to AB, EF is parallel to BC, and DF is parallel to AC. To prove: $\triangle ABC$ is similar to $\triangle DEF$.

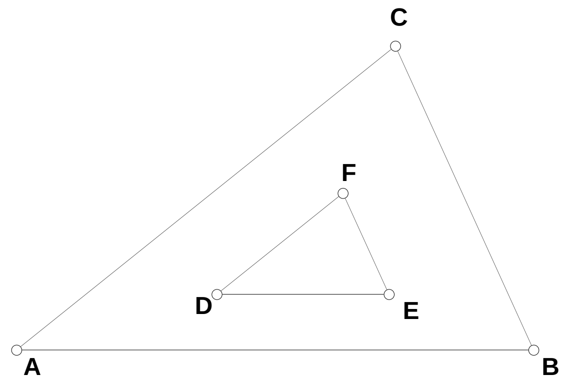

Figure 4

9. Let ABC be a triangle and let D and E be points on the segments AC and BC, respectively, with DE parallel to AB. Let M be the midpoint of AB, and let N be the intersection of DE and CM. Prove that N is the midpoint of DE. (Hint: Use two pairs of similar triangles).

10. (See Figure 5.) For this problem you need the definition of circle: a *circle* consists of all of the points which are at a given distance (called the *radius*) from a given point (called the *center*).

    Given: O is the center of the circle. To prove: $\angle AOC = 2\angle ABC$. (Hint: Use algebra as one ingredient in your proof.)

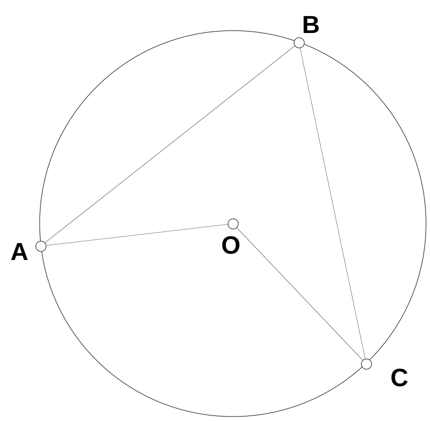

Figure 5