Computer Project # 3

Predator-Prey Equations

Goal: Investigate the qualitative behavior of a *nonlinear* system of differential equations.

Tools needed: pplane7.

Description: A farmer has ladybugs and aphids in her fields. The helpful ladybugs (predator) eat the destructive aphids (prey) who devour her crops.

Let $\boldsymbol{x(t)} =$ aphid population (in millions) at time t
$\boldsymbol{y(t)} =$ ladybug population (in millions) at time t

The farmer knows that the growth rates of the aphid and ladybug populations are given by

$$\begin{cases} \dfrac{dx}{dt} = x\,(1-y) \\ \\ \dfrac{dy}{dt} = y\,(x-1) \end{cases} .$$

Questions: Assume there are initially 800,000 aphids and 400,000 ladybugs in what follows below.

(1) Use **pplane6** to plot the trajectory through (0.8, 0.4). As t increases, describe what happens to each population. Is the aphid population ever smaller than 300,000 ? Are the aphids ever eradicated ? Does the ladybug population ever exceed 2 million ?

(2) A fellow farmer suggests that she use pesticide to kill the aphids. She is reluctant because it also kills the helpful ladybugs and she prefers to have some ladybugs remaining to eat other destructive insects. If she were to use a pesticide, the growth rates would then become

$$(*) \quad \begin{cases} \dfrac{dx}{dt} = x\,(1-y) - s\,x \\ \\ \dfrac{dy}{dt} = y\,(x-1) - s\,y \end{cases}$$

where $s \geq 0$ is a measure of the "strength" of the pesticide - the larger the s, the stronger the pesiticide. Currently there are only two commerically available strengths : $s = 0.5$ and $s = 0.75$. Plot the trajectories for the new system of equations $(*)$ with these values of s. Will the aphids ever be totally eliminated ?

(3) If she knows her crops will survive if the aphid population never exceeds 2.6 million, which strength (if any) would you recommend she use : $s = 0.0$ (no pesticide), $s = 0.5$, $s = 0.75$?

(4) By special permission, she could get a pesticide with the maximum strength of $s = 1.5$. Plot this trajectory. What happens to the ladybugs and aphids if she uses this pesticide ?