MATLAB.6

Numerical Methods
First consider the ode

```
y'=f(t,y)=2*t*y^2,
y(0)=0.1.
```

We analyze this using the Euler, Improved Euler and Runge-Kutta methods.
First we make an M-file for the right hand side.

```
*******************************************
function w=f(t,y)
w=2*t*y^2;
*******************************************
```

Next save the M-files (eul.m, rk2.m, and rk4.m) for each of the three methods.

The syntax for the Euler method (eul.m) is `[t,y]=eul(@f,tspan,y0,stepsize)`,

where f is the name of the function m-file, tspan is the vector [t0,tfinal] containing the initial and final time conditions, y0 is the value of the initial condition, and stepsize is the step size, h, used in the method.
The syntax for the Improved Euler method is `[t,y]=rk2(@f,tspan,y0,stepsize)`.
The syntax for the Runga-Kutta method is `[t,y]=rk4(@f,tspan,y0,stepsize)`.

To approximate the solution to the above equation with the various methods over the interval [0,3] with step size h=3/20.

```
************************************************
[teul,yeul]=eul(@f,[0,3],0.1,3/20);
[trk2,yrk2]=rk2(@f,[0,3],0.1,3/20);
[trk4,yrk4]=rk4(@f,[0,3],0.1,3/20);
plot(teul,yeul,trk2,yrk2,'--',trk4,yrk4,'o')
*************************************************
```

Euler is given by a solid curve,Improved Euler by '--'s and
Runge-Kutta by 'o's.

ASSIGNMENT 6:

1. Let f(t,y) be as above. Write an m-file for f. Plot the approximate solutions to the above differential equation given by eul, rk2, and rk4 on the interval [0,3] with step size 3/20.

The actual solution to the differential equation is y(t)=1/(10-t^2).
We will examine how accurate our approximations are. To find the distance between y(t) and the Euler approximation on the interval [0,3] for a given step size h type:

```
*****************************
t=0:h:3;
y=1./(10-t.^2);
y=y';
% Here we have transposed y from a row vector to a column vector.
[teul,yeul]=eul(@f,[0,3],0.1,h);
max(abs(y-yeul))

*****************************
```

Here

```
 abs(y-yeul)=[abs(y(1)-yeul(1)),.., abs(y(3/h)-yeul(3/h))]'
```

and max(abs(y-yeul)) is the maximum of the 3/h components.

The theory predicts that

```
(I)                 max(abs(y-yeul))<C*(h)
```

for some constant C and each h.

2. Set C1(h) = max(abs(y-yeul)) / h. Compute C1(h) for h=.01, h=.001, h=.0001, and h=.00001.
Does C1(h) grow as h gets small or tend to level off?

Is this consistent with (I)?

3. Set C2(h)=max(abs(y-yrk2)) / h^2 for the Improved Euler method.
Compute C2(h) for the same values of h.
What should C3(n) be for the Runge-Kutta method? Compute C3(h).