Lesson 39 Appendix I Section 5.6 (part 1)

Any of you who are familiar with financial plans or retirement investments know about annuities.

An annuity is a plan involving payments made at regular intervals. An **ordinary annuity** is one in which the payments are made at the end of each time interval. In this lesson, we will be discussing ordinary annuities.

The **future value** of an annuity is the **sum of all the payments and the interest those payments earn.** Suppose a person makes a payment of \$500 every 3 months for 20 years. The amount of money in that account at the end of the 20 years is the future value of the annuity.

Formula for the Future Value of an Annuity: The future value S of an ordinary annuity with deposits or payments of R made regularly k times per year for t years, with interest compounded k times per year at an annual rate r, is given by…

$$S = R\left[\frac{(1+i)^{kt}-1}{i}\right], \text{ where } i = \frac{r}{k}$$

Note: The frequency of compounding per year always equals the types of payments. For example, if a person makes monthly payments, then the interest is compounded monthly. If a person makes payments every 6 monthly, then the interest is compounded semiannually.

<u>Ex 1:</u> Assume that \$1200 is deposited at the end of each year into an account in which interest is compounded annually at a rate of 5%. Find the accumulated amount (future value) after 6 years.

<u>Ex 2:</u> Assume that \$1200 is deposited every 3 months into an account in which interest is compounded quarterly 3 ½ % annual interest. Find the accumulated amount (future value) after 8 years.

Ex 3: Assume that \$1200 is deposited monthly in an account in which interest is compounded monthly at an annual rate of 8%. Find the accumulated amount (future value) after 12 years.

If the situation of an ordinary annuity is reversed and we need to find what **regular deposit or payment** should be made in order to provide a specific future amount, the following formula would be used. This amount R is called a sinking fund payment. Suppose Karen wants to know how much she should deposit monthly in order to have \$20,000 in 10 years. This is the type of situation for the following formula.

Sinking Fund Payment: For an annuity to provide a future value S, regular deposits R are made k times per year for t years, with interest compounded k times a year at an annual interest rate r. The payment R is given by….

$$R = \frac{Si}{(1+i)^{kt} - 1}, \text{ where } i \text{ (the periodic rate)} = \frac{r}{k}$$

Ex 4: Find the amount of each regular payment that would provide \$30,000 in 15 years at an annual rate of 6% and compounding semiannually.

Ex 5: A teacher has been making monthly payments of $350 into a retirement account. This account earns 3.5% annual interest and is compounded monthly. If she retires in 20 years after she started making payments, how much will be in the account?

Ex 6: A mother began putting $1250 every 3 months in an college account for her daughter beginning when the daughter turned 12 years old. If the account earns 8 ½ % annual interest, compounded quarterly, how much is in the account on the daughter's 20th birthday?

Ex 7: Suppose Gail want to retire with $100,000 in an annuity in 20 years time. If she can invest at 4% compounded monthly, how much should she put in the retirement fund each month?

Ex 8: Roger wants to retire in 40 years. He plans on investing \$2000 every 6 months for 20 years. Then he will let the accumulated money continue to grow for the remaining 20 years before retirement. Assume the account earns 6% compounded semiannually. How much will Roger have when he retires?

Ex 9: Roger's brother, Ryan, also wants to retire in 40 years. However, he has a different plan than Roger. He wants to wait 20 years and then invest twice as much as Roger every 6 months, \$4000 for 20 years. Assume his account also earns 6% compounded semiannually. How much will Ryan have when he retires?

Suppose this question is asked, 'What single payment now or single amount now will provide the same future amount as an annuity?' This is called **the present value of an annuity.** A typical real life situation where the present value of an annuity can be understood would involve lottery winnings. Many people who win a lottery want a sum of money right now, rather than receiving regular payments over a period of time. For example, suppose Jon wins a lottery and is to receive \$2000 monthly for 15 years. He may decide he would rather have a one time payment right now. Of course, that one time payment will be less than the total he would receive by totaling the regular payments over 15 years. But many people prefer the 'up front' amount so they can invest on their own or use the money right away.

Present Value of an Annuity: The present value P of an annuity with payment of R dollars made k times per year for t years, with interest compounded k times per year at an annual rate r is

$$P = R\left[\frac{1-(1+i)^{-kt}}{i}\right]$$

where $i = \frac{r}{k}$, the periodic rate.

Ex 10: Julie wants to save some money for college. She is willing to save \$50 a month in an account earning 8% interest for 5 years, compounded monthly.

a) How much money would she deposit over the 5 years?

b) How much will be in the account at the end of the 5 years?

c) What single deposit now would provide the same amount at the end of the 5 years? Is this a better deal for Julie? Why or why not?

Ex 11: Find the present value of an annuity for an account with semiannual payments of \$375 at a 4.92% annual rate, compounded semiannually for 10 years.

Ex 12: Instead of making quarterly contributions of \$700 to a retirement fund for the next 15 years, a man would rather make only one contribution now. How much should that be? Assume 6 ¼ % annual interest, compounded quarterly.

Ex 13: Instead of receiving an annuity of \$12,000 each year for the next 15 years, a young woman would like a one-time payment, now. Assuming an annual rate of 8.5% compounded annually, what would be a fair amount?