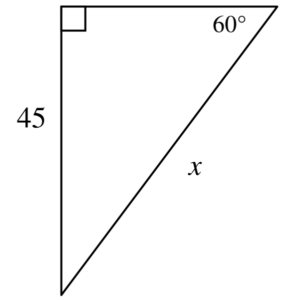
1) Given the right triangle at the right, find the **exact** length of the side labeled *x*. <u>Rationalize the denominator, if necessary.</u>



A $45\sqrt{2}$

B $\dfrac{45\sqrt{3}}{2}$

C 90

D $30\sqrt{3}$

E $\dfrac{45\sqrt{2}}{2}$

$$\sin(60^\circ) = \frac{opp}{hyp} = \frac{45}{x}$$
$$\frac{\sqrt{3}}{2} = \frac{45}{x}$$
$$x\sqrt{3} = 90$$
$$x = \frac{90}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{90\sqrt{3}}{3} = 30\sqrt{3}$$

2) Find the approximate value of $\sec\left(\frac{156\pi}{5}\right)$ to 4 decimal places. Check the **mode** on your calculator.

A -1.2361

B -0.1395

C -0.8090

D -7.1695

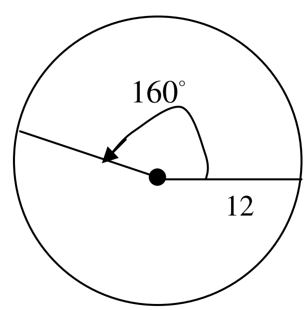
E 1.0099

$$\sec\left(\frac{156\pi}{5}\right) = \frac{1}{\cos\left(\frac{156\pi}{5}\right)}$$

Put calculator in radian mode.

$$= \frac{1}{\cos(98.01769079)} = \frac{1}{-0.809016994}$$
$$\approx -1.2361$$

3) Find the **area of the sector** formed by the central angle shown. Approximate to the nearest tenth if necessary.



A 11,520 square units

B 1440 square units

C 201.1 square units

D 100.5 square units

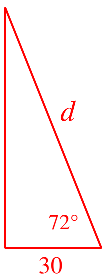
E 402.1 square units

$$A = \tfrac{1}{2} r^2\theta$$

θ must be in radians

$$160^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{8\pi}{9}$$
$$A = \tfrac{1}{2}(12)^2\left(\frac{8\pi}{9}\right)$$
$$A = \frac{1}{2}(144)\left(\frac{8\pi}{9}\right) = 8(8\pi) = 64\pi$$
$$A \approx 201.1$$

4) Kyle is standing on level ground 30 feet from the base of a flagpole. He finds the angle of elevation from his feet to the top of the pole is 72°. Approximate the **distance from Kyle's feet to the top of the flagpole** to the nearest tenth of a foot.



A 31.5 *ft.*
B 92.3 *ft.*
C 63.0 *ft.*
D 97.1 *ft.*
E None of the above.

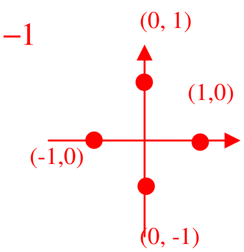
$$\cos(72^\circ) = \frac{30}{d}$$
$$d\left(\cos(72^\circ)\right) = 30$$
$$d = \frac{30}{\cos(72^\circ)} = \frac{30}{0.309016994} \approx 97.1$$

5) Which statement(s) below is(are) **true**?

I	$\cos\pi = -1$
II	$\tan\left(-\frac{\pi}{2}\right)$ is undefined
III	$\csc\left(\frac{3\pi}{2}\right) = 1$

A I and III only
B II and III only
C I only
D II only
E I and II only

These are all quadrantal angles.
$\cos\pi = -1$
$\tan\left(-\frac{\pi}{2}\right) = \tan\left(\frac{3\pi}{2}\right)$ is undefined (zero den.)
$\csc\left(\frac{3\pi}{2}\right) = \frac{1}{\sin\left(\frac{3\pi}{2}\right)} = \frac{1}{-1} = -1$



Answer: I and II only

6) Find the exact value of $\cot\left(\frac{5\pi}{6}\right)$.

A $\frac{1}{\sqrt{3}}$
B $-\frac{1}{\sqrt{3}}$
C -1
D $\sqrt{3}$
E None of the above.

$$\cot\left(\frac{5\pi}{6}\right) = \frac{1}{\tan\left(\frac{5\pi}{6}\right)}$$

$\frac{5\pi}{6}$ is a 30° angle in QII, tangent is negative

$$= \frac{1}{-\tan\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{-1}{\sqrt{3}}} = -\sqrt{3}$$

Answer is not listed; none of the above.

7) Approximate to the nearest tenth of a degree, all angles θ in the interval [0°, 360°) that satisfy the equation $\sin\theta = -0.294$. Check mode on calculator.

A $\theta = 197.1^\circ$, $\theta = 342.9^\circ$

B $\theta = 107.1^\circ$, $\theta = 287.1^\circ$

C $\theta = 17.1^\circ$, $\theta = 162.9^\circ$

D $\theta = 72.9^\circ$, $\theta = 252.9^\circ$

E $\theta = 162.9^\circ$, $\theta = 197.1^\circ$

Put calculator in degree mode. Use the value 0.294 and the 2nd and sine keys to find the reference angle.

$\theta_R = 17.1^\circ$

The sine is negative in Q III and IV.

$\theta = 180^\circ + 17.1^\circ = 197.1^\circ$

and $\theta = 360^\circ - 17.1^\circ = 342.9^\circ$

8) Which statement below is **false**?

A Angles $\frac{5\pi}{6}$ and -210° are coterminal angles.

B Angles with measurements of $25^\circ 12'$ and 64.8° are complementary.

C If $\theta = 980^\circ$, then the reference angle $\theta_R = 10^\circ$.

D If the $\cos\theta > 0$ and $\tan\theta < 0$, then θ is in quadrant IV.

E Given point $\left(-\frac{3}{5}, \frac{4}{5}\right)$ that is $P(t)$ on a unit circle, then $P(-t)$ is $\left(-\frac{3}{5}, -\frac{4}{5}\right)$.

A Convert both to between 0 and 360 degrees: $\frac{5\pi}{6} = \frac{5\pi}{6}\left(\frac{180^\circ}{\pi}\right) = 150^\circ \quad -210^\circ + 360^\circ = 150^\circ$

Both are equivalent to 150°; are coterminal

B Convert to minutes to a decimal, add the two angle measurements: $12' = 12'\left(\frac{1^\circ}{60'}\right) = 0.2$

$25.2^\circ + 64.8^\circ = 90.0^\circ$

Angles are complementary, since they have a sum of 90°.

C Get θ between 0 and 360 degrees: 980° - 2(360°) = 260°

$\theta_R = 260^\circ - 180^\circ = 80^\circ$ **This is the false statement.**

D The cosine is positive in Q I and IV. The tangent is negative in Q II and IV. Therefore, the quadrant for both conditions is Q IV.

E The given point $P(t)$ is in Q II. The negative of t would be in Q III; both coordinates should be negative.

9) Which statement below is **false** about the graph of $y=\frac{2}{3}\sin\left(2x+\frac{\pi}{2}\right)$?

A The phase shift is $\frac{\pi}{4}$.

B The graph crosses the y-axis at $\frac{2}{3}$.

C The amplitude is $\frac{2}{3}$.

D The period is π.

E One place where the graph crosses the x-axis is $\frac{3\pi}{4}$.

The amplitude of the graph is $|a|=\frac{2}{3}$. The period is $\frac{2\pi}{|b|}=\frac{2\pi}{2}=\pi$.

The phase shift is $-\frac{c}{b}=\frac{-\frac{\pi}{2}}{2}=-\frac{\pi}{4}$. Sketch the graph of the function. The y-intercept is 2/3. There are x-intercepts at $\frac{\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}$, and $-\frac{3\pi}{4}$.

The false statement is the one about the phase shift.

10) Given a point $P(t)=\left(\frac{3}{4}, -\frac{\sqrt{7}}{4}\right)$ on a unit circle, find the value of $\sec t$ and $\tan t$.

A $\sec t=\frac{-4}{3}$, $\tan t=\frac{-\sqrt{7}}{3}$

B $\sec t=\frac{4}{3}$, $\tan t=\frac{\sqrt{7}}{4}$

C $\sec t=\frac{3}{4}$, $\tan t=\frac{-3}{\sqrt{7}}$

D $\sec t=\frac{4}{3}$, $\tan t=\frac{-\sqrt{7}}{3}$

E $\sec t=\frac{-4}{3}$, $\tan t=\frac{-3}{\sqrt{7}}$

The x coordinate is the cosine of t and the y coordinate is the sine of t.

$$\sec t=\frac{1}{\cos t}=\frac{1}{\frac{3}{4}}=\frac{4}{3}$$

$$\tan t=\frac{\sin t}{\cos t}=\frac{\frac{-\sqrt{7}}{4}}{\frac{3}{4}}=\frac{-\sqrt{7}}{3}$$

11) $\dfrac{\cos(-x) - \csc(-x)}{\cos x}$ is equivalent to which of the following? **Hint:** Begin with the negative formulas.

A $1+\csc x\cos x$

B $1+\csc x\sec x$

C $\csc x\sec x-1$

D $1-\csc x\sec x$

E $\sin x\sec x-1$

$$\frac{\cos(-x)-\csc(-x)}{\cos x}=\frac{\cos x-(-\csc x)}{\cos x}$$
$$=\frac{\cos x}{\cos x}+\frac{\csc x}{\cos x}$$
$$=1+\csc x\left(\frac{1}{\cos x}\right)$$
$$=1+\csc x\sec x$$

12) What is the period of the graph below?

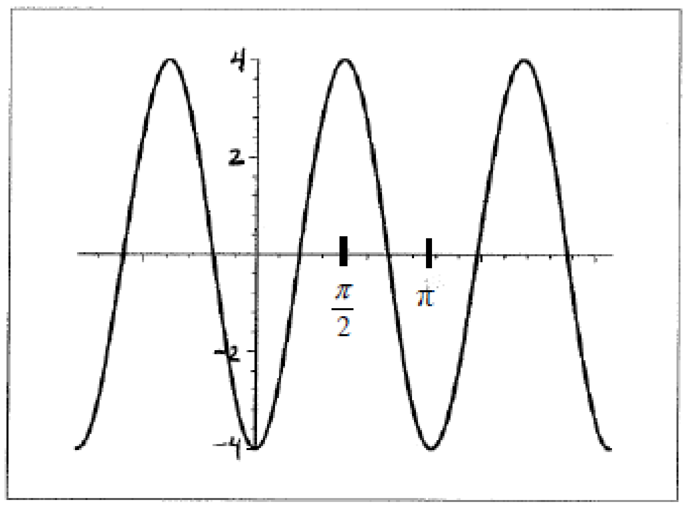
A $\frac{\pi}{2}$

B π

C $\frac{5\pi}{4}$

D $\frac{3\pi}{2}$

E $\frac{3\pi}{4}$



A period can be found from $\frac{\pi}{4}$ (halfway between 0 and $\frac{\pi}{2}$) to $\frac{5\pi}{4}$ ($\frac{\pi}{4}$ right of π). That equals π radians. Period is π.

13) Find the equation, in the form $y = a\sin(bx + c)$, where $a > 0$, $b > 0$, and c is the least positive real number for a sine curve with a period of 4, an amplitude of 2, and a phase shift of -1.

A $y = 2\sin(4x + 1)$

B $y = 2\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

C $y = 2\sin(\pi x + \pi)$

D $y = 2\sin\left(\frac{\pi}{4}x + \frac{\pi}{4}\right)$

E $y = 2\sin\left(\frac{\pi}{2}x + 2\pi\right)$

$a = 2$ (amplitude)

Period: $\frac{2\pi}{b} = 4 \to 4b = 2\pi \to b = \frac{\pi}{2}$

Phase Shift:

$$-\frac{c}{b} = \frac{-c}{\frac{\pi}{2}} = \frac{-2c}{\pi} = -1 \to -2c = -\pi \to c = \frac{\pi}{2}$$

$$y = 2\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$$

14) Find the exact value of $\sec\theta$ if θ is in standard position and the terminal side of θ is in quadrant II and is parallel to the line $5x + 2y = 7$.

A $\sec\theta = \frac{-\sqrt{29}}{2}$

B $\sec\theta = \frac{-\sqrt{29}}{5}$

C $\sec\theta = \frac{\sqrt{29}}{2}$

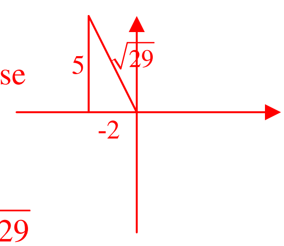
D $\sec\theta = \frac{-5}{\sqrt{29}}$

E $\sec\theta = \frac{-2}{\sqrt{29}}$

The slope of the line = the tangent of the angle.

$m = -\frac{5}{2} = \tan\theta = \frac{opp}{adj}$

The length of the hypotenuse can be found using the Pythagorean theorem.



$\cos\theta = \frac{-2}{\sqrt{29}} \to \sec\theta = \frac{-\sqrt{29}}{2}$

The answer could also be found using the identity $1 + \tan^2\theta = \sec^2\theta$.

15) Which statement(s) below is(are) **true** about the graph of $y = \cos x - 3$?

I	The graph contains the point $\left(-\frac{\pi}{2}, -3\right)$.
II	The graph crosses the y-axis at -2.
III	The graph never crosses the x-axis.

A I and II only
B I and III only
C II and III only
D II only
E I, II, and III

This is a basic cosine graph shifted 3 down. The basic graph would contain the point $\left(\frac{-\pi}{2}, 0\right)$. Shift down 3, you get $\left(\frac{-\pi}{2}, -3\right)$. **Statement I is true**. The range is [-4, -2]. Since the cosine graph begins at the maximum height, the y-intercept will be -2. **Statement II is true**. Also, from the range, it can be determined that the graph never crosses the x-axis. **Statement III is true.**
I, II, and III