MA 108

Math—The Language of Engineers

by

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Let Me Introduce Myself to You

- https://engineering.purdue.edu/ECE/People/

Outline

- What is math?

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- Some pure math problems

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- Some cool math applications
- Conclusions

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- “Mathematical discoveries have come both from the attempt to describe the natural world and from the desire to arrive at a form of inescapable truth from careful reasoning”—Kenyon College Math Department Web Page

Pure Math vs. Applied Math

- The Millennium Prize Problems

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- http://www.claymath.org/millennium/

Engineering Grand Challenges

- 14 grand challenges for engineering in the 21-st century identified by the National Academy of Engineering

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- What does “best” mean?

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- $\Omega =$ set of all possible choices (*Feasible set*)

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- f is a real-valued function on n variables,

$$f : \mathbb{R}^n \longrightarrow \mathbb{R}$$

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- Numerically

Example: Linear regression

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$$(t_0, y_0), \ldots, (t_n, y_n)$$

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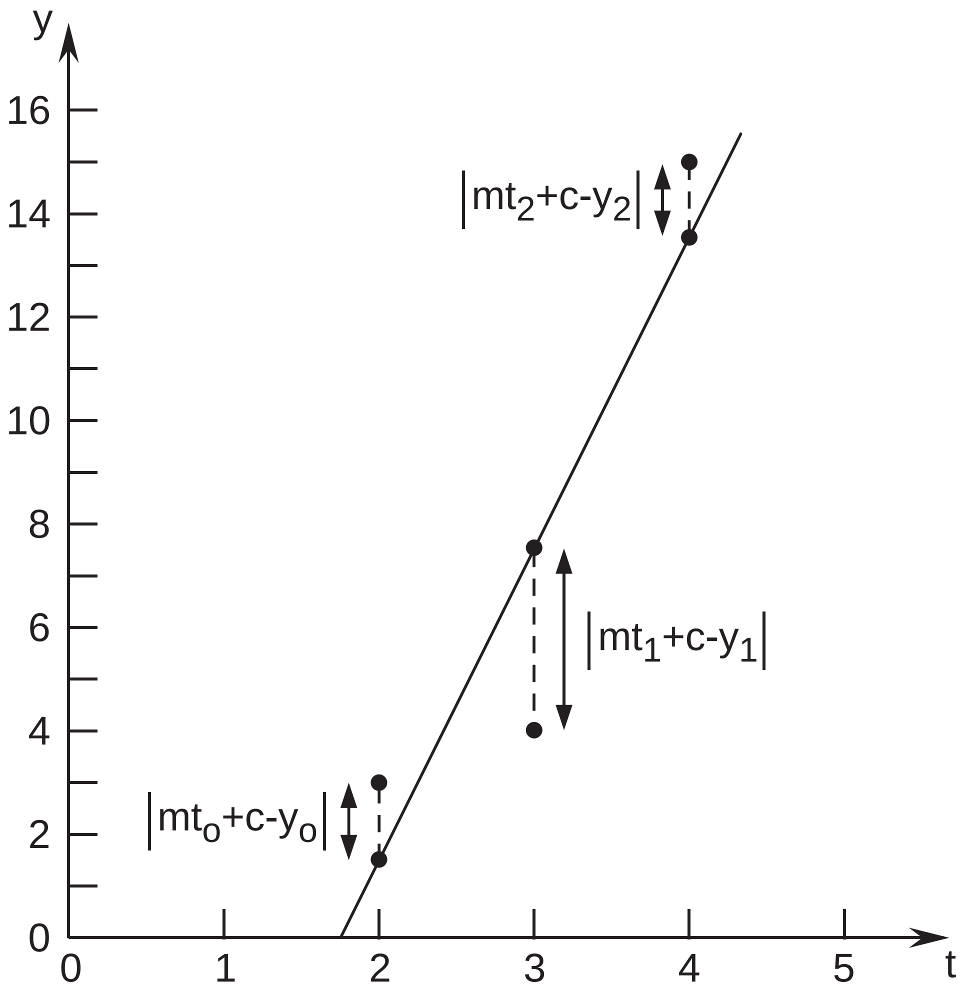
- Best = minimize the average squared error

Minimizing the average squared error



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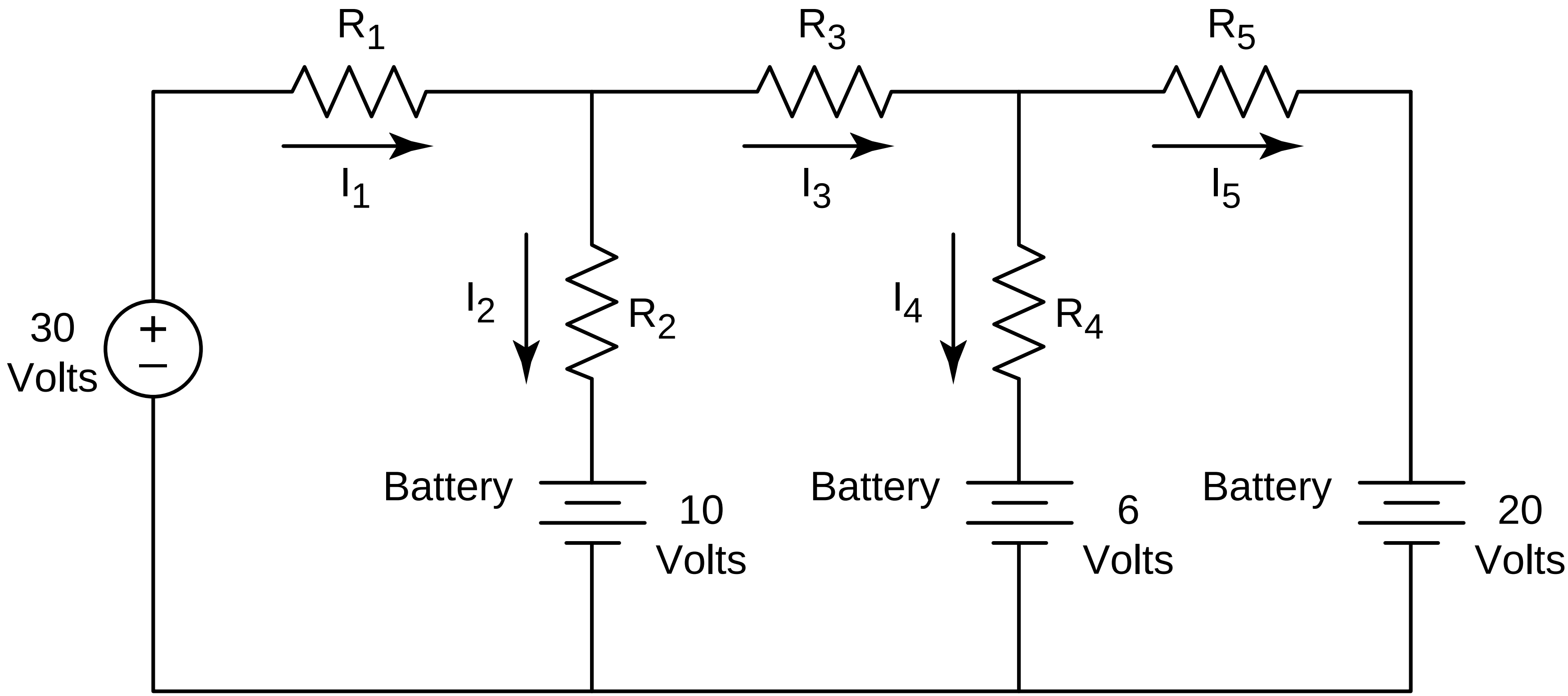
- Related application: system identification

Battery charger circuit



Charger circuit specifications

Current	I_1	I_2	I_3	I_4	I_5
Upper Limit (Amps)	4	3	3	2	2
Lower Limit (Amps)	0	0	0	0	0

Design objective

Find $I_1, \ldots, I_5$ to maximize power transferred to batteries, that is,

$$\begin{aligned} \max \quad & 10I_2 + 6I_4 + 20I_5 \\ \text{subject to} \quad & I_1 = I_2 + I_3 \\ & I_3 = I_4 + I_5 \\ & I_1 \leq 4 \\ & I_2 \leq 3 \\ & I_3 \leq 3 \\ & I_4 \leq 2 \\ & I_5 \leq 2, \\ & I_1, I_2, I_3, I_4, I_5 \geq 0 \end{aligned}$$

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- Solution: Can use the simplex algorithm—see MA 511

Model-based Predictive Control

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- The idea behind this approach can be explained using an example of driving a car

MPC—analogy with driving a car

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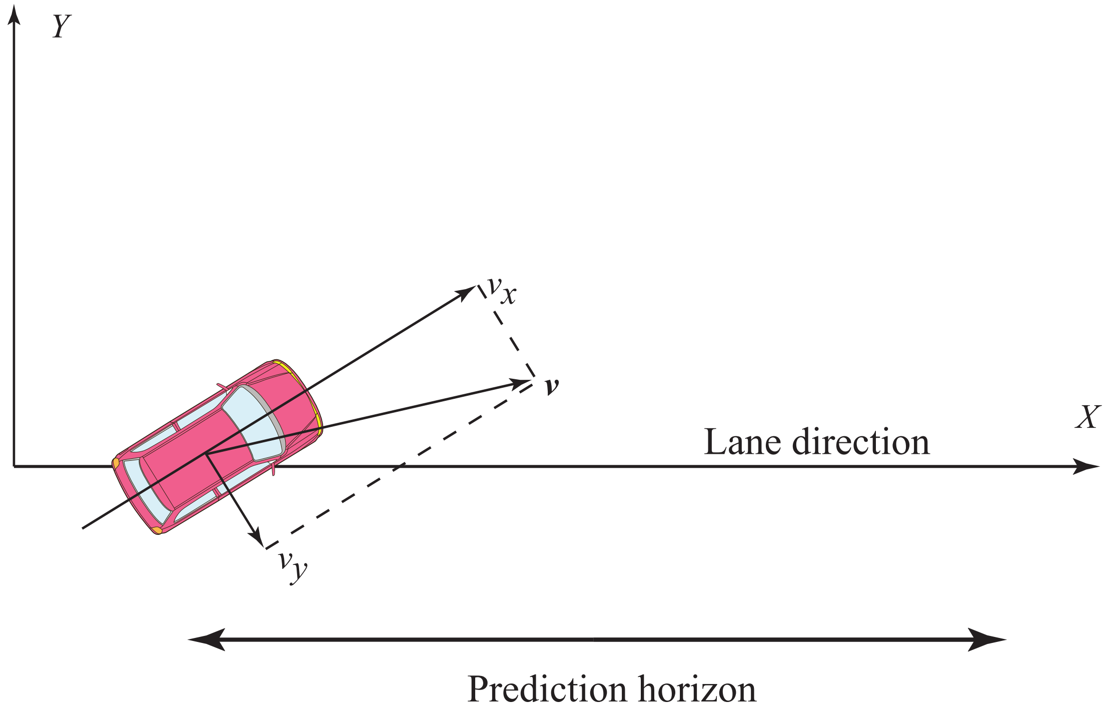
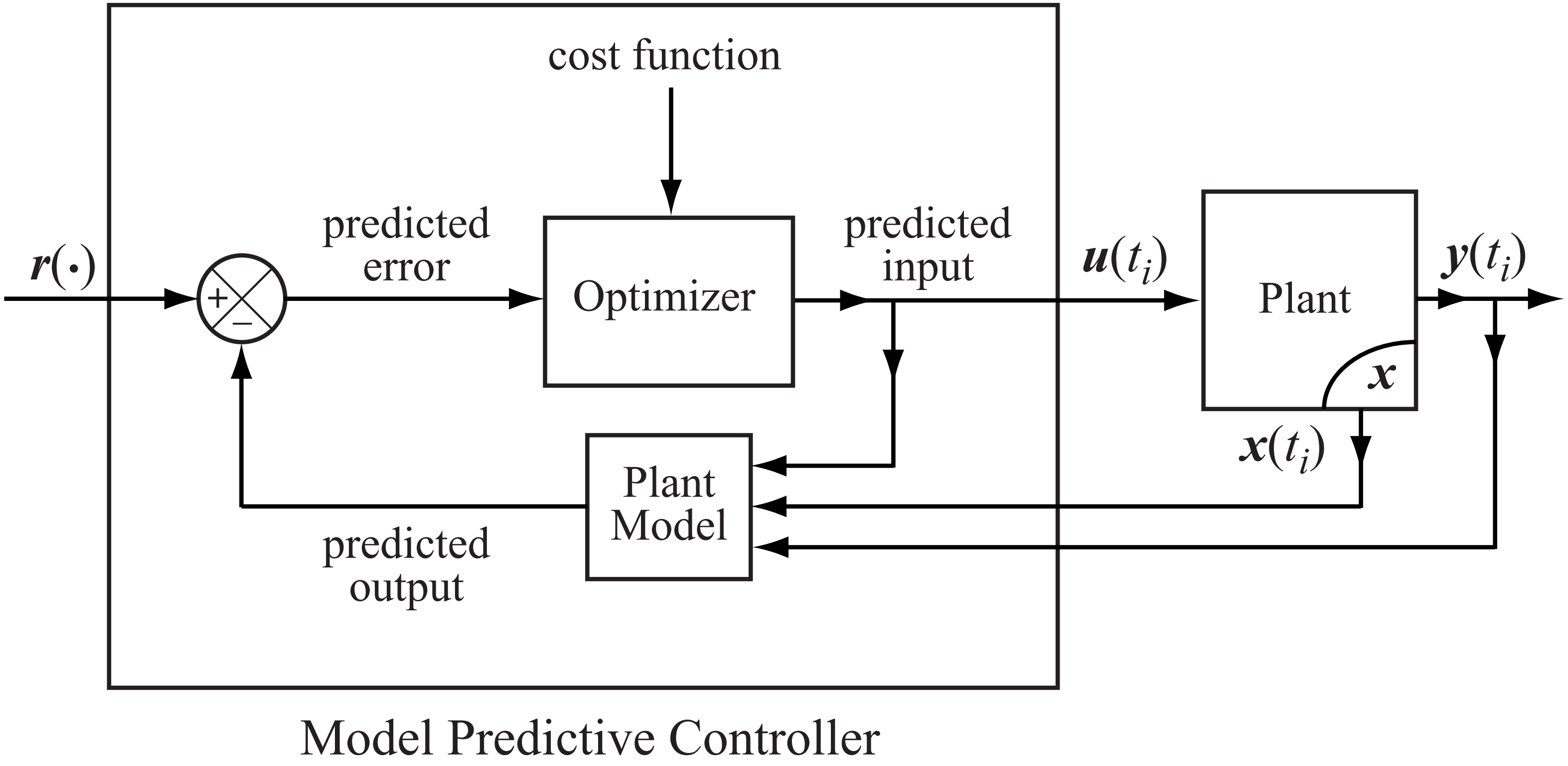
- Based on the prediction, the driver adjusts the driving direction

MPC illustration



The driver predicts future travel direction based on the current state of the car and the current position of the steering wheel

Basic Structure of MPC



Hypothalamic-pituitary-adrenal axis

- The hypothalamic-pituitary-adrenal—HPA

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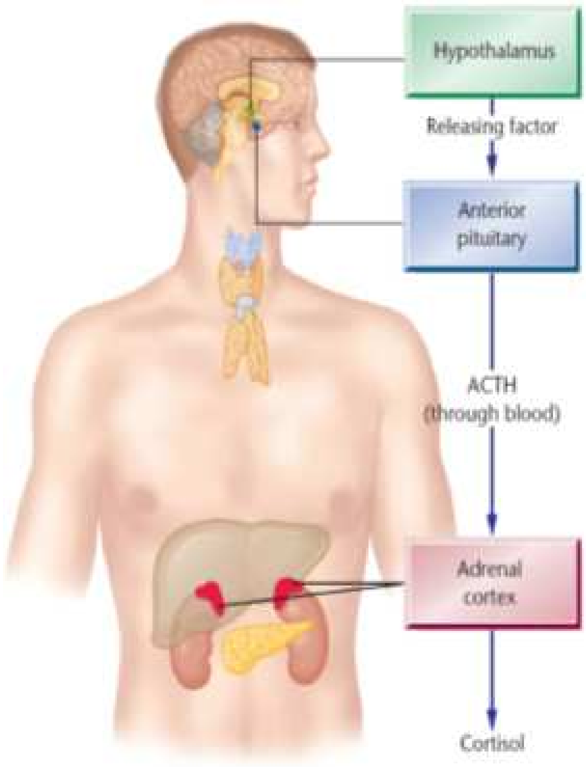
- The HPA axis is a set of interactions between the hypothalamus (a part of the brain), the pituitary gland (also part of the brain) and the adrenal or suprarenal glands (at the top of each kidney)

Biology of the HPA axis



http://www.vitalifenetworks.com/images/Cortisol_image.jpg

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- It is also a major part of the system that controls our reaction to stress, trauma and injury

HPA axis therapeutic correction

- A problem related to human health—how optimization can be used to find a therapeutic strategy

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- http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2613

Computational Aspects

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- MATHEMATICA

Math and Soccer?

- http://www.bbc.co.uk/news/science-environment-11153466

Conclusions

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- Math is powerful
- Can do cool things using math
- Need to consider the moral consequences of what we do!